THE UNIVERSITY OF HONG KONG

*Institute of Mathematical Research*
*Department of Mathematics*

# "MATH for Machine Learning" Seminar

# Convergence of difference inclusions: a diameter criterion and step-size conditions


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## Abstract

We study bounded realizations of discrete difference inclusions with set-valued increments and additive errors. Our results have two parts. First, we give a general convergence criterion in which changes in a convergent scalar quantity control the diameters of local sequence segments near an accumulation point. We also develop a stratified descent framework for verifying this control. For convergent realizations, the framework yields a stationarity condition for the limit from outer limits of the scaled update map. When applied to first-order methods for minimizing locally Lipschitz objectives definable in polynomially bounded o-minimal structures, the framework yields convergence to critical points for bounded sequences generated by the inexact subgradient, momentum, and stochastic subgradient methods with step sizes of order $k^{-1}$, under the corresponding error and noise conditions.

Second, we study first-order methods with polynomial step sizes of order $k^{-a}$. At the square-summability boundary $a = 1/2$, we construct a locally Lipschitz semialgebraic objective for which the exact subgradient method generates a bounded, nonconvergent sequence whose accumulation points satisfy our active-geometry assumptions. Under these assumptions, bounded sequences generated by the momentum method converge for $1/2 < a \leq 1$, while bounded sequences generated by the stochastic subgradient method converge almost surely for $2/3 < a \leq 1$. The momentum range is sharp for this method class, while the stochastic range matches recent full-sequence convergence results for smooth nonconvex objectives.


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| Date: | September 17, 2026 (Thursday) |
| Time: | 3:00 pm – 4:00 pm |
| Venue: | Room 210, Run Run Shaw Building HKU |

*All are welcome*