



## Numerical Analysis Seminar

### A strongly mass-conservative method for the coupled Navier-Stokes and Darcy-Forchheimer equations

Mr. Jingyu LIU  
City University of Hong Kong

#### Abstract

In this talk, I will present a strongly mass-conservative numerical method for a coupled Navier-Stokes and Darcy-Forchheimer system, which models the interaction between free fluid flow and porous media flow. These two subproblems are coupled through physically relevant interface conditions, including mass conservation, balance of normal forces, and the Beavers-Joseph-Saffman condition.

The proposed scheme combines a staggered discontinuous Galerkin method for the Navier-Stokes equations with standard mixed finite element method for the Darcy-Forchheimer problem. A key feature of the formulation is that the interface conditions are incorporated directly into the scheme, without the use of Lagrange multipliers. As a result, the method preserves incompressibility in the free-flow region and achieves strong mass conservation across the entire coupled domain.

Under suitable small-data assumptions, we establish well-posedness and derive optimal-order convergence estimates for all variables. A careful treatment of the interface terms is essential to the analysis. As a consequence of strong mass conservation, the velocity and velocity gradient errors are independent of the pressure, demonstrating the pressure-robustness of the method. Moreover, the error bounds do not involve negative powers of the viscosity, ensuring robustness in the small-viscosity regime.

Date: January 14, 2026 (Wednesday)
Time: 3:00 pm - 4:00 pm
Venue: Room 210, Run Run Shaw Building, HKU