THE UNIVERSITY OF HONG KONG

Department of Mathematics

Qualifying Research Seminar

First-Order Dynamics in O-Minimal Structures: Convergence, Geometry, and Criticality

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Rm 210, Run Run Shaw Building, HKU

Abstract

First-order methods are among the most fundamental tools in optimization, learning, and nonsmooth dynamical systems. In nonconvex and nonsmooth settings, however, their discrete trajectories can exhibit highly subtle behavior: even when function values converge and cluster points are critical, the iterates themselves may still oscillate, and the discrete dynamics can differ substantially from their continuous-time limits. This makes it a central problem to understand which geometric structures ensure convergence, control oscillation, and describe asymptotic behavior of first-order dynamics.

In this talk, I will focus on first-order dynamics for functions definable in polynomially bounded o-minimal structures. This setting provides a broad class of tame nonsmooth problems while retaining enough geometric structure to support a refined asymptotic analysis. I will discuss several guiding questions: when does a bounded first-order sequence converge, what mechanism controls its global oscillation, how can one characterize its limit points, and what kind of local behavior should be expected near critical points.

I will then present two recent works in this direction. The first shows that, for subgradient sequences, the diameter of the discrete trajectory can be controlled by the variation of function values together with higher-order step-size terms, leading in particular to convergence of bounded subgradient sequences under step sizes of order $1/k$. The second develops a more general framework for difference inclusions, based on a diameter criterion for convergence and a stratified descent mechanism, which extends the analysis to inexact and stochastic subgradient methods as well as momentum-type schemes, and also yields a criticality characterization of limit points. These results provide a purely discrete approach to convergence, without relying on continuous-time approximation arguments. Finally, I will discuss ongoing and future directions, especially the study of subgradient dynamics near critical points, where one expects finer geometric and dynamical descriptions of the asymptotic regime.

All are welcome