THE UNIVERSITY OF HONG KONG

Department of Mathematics

Qualifying Research Seminar

Manifold Optimization with Linear Constraints on Hilbert Spaces: Theoretical Foundation, Algorithmic Design, and Industrial Applications

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Rm 210, Run Run Shaw Building, HKU

Abstract

To address complex applications such as the simulation problems of liquid crystals and elastic rods in industry, we need to solve optimization problems with linear constraints over infinite-dimensional manifolds in Hilbert spaces. These problems are theoretically and numerically challenging because of the infinite-dimensional nature of their configuration spaces, the non-convexity of the underlying manifold constraints, and the presence of additional linear constraints. Essentially, there is no mature theory or efficient algorithm in the literature.

We plan to initiate the foundation of theoretical analysis and algorithmic design for this class of problems and solve some real applications in industry. More specifically, we plan to discuss how to embed the linear constraints into a manifold, thereby transforming the problem into an equivalent unconstrained formulation on the intersection of the manifold. We will then establish the well-posedness and characterize the geometric properties of the embedded manifold, including the tangent spaces. Furthermore, we construct implicit local retraction and projection operators using techniques from functional analysis, and derive the weak form of the

Riemannian Fréchet derivatives for the objective functional. Building upon this theoretical foundation, we launch the first framework of algorithmic design with non-orthogonal projection operators alongside weak forms of the gradient and Hessian, and analyze the convergence rigorously. In particular, we will focus on preconditioning strategies that bypass the expensive calculation of linear systems required for the true Riemannian gradient. Furthermore, we will apply the algorithms to solve some real problems in industry, such as the large-displacement computation of inextensible flexible rods. We target an original and systematic study of this class of problems from the perspectives of theoretical foundation, algorithmic design, and implementations to real industrial applications.

All are welcome