THE UNIVERSITY OF HONG KONG

Department of Mathematics

Qualifying Research Seminar

Efficient Numerical Algorithms for Optimal Control of Large-Scale Multi-Agent Systems

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March 6, 2026 (Friday) at 4:30pm

Rm 210, Run Run Shaw Building, HKU

Abstract

The study of multi-agent systems—comprising large groups of interacting individuals such as flocking birds, schooling fish, or human crowds—has become a cornerstone of modern research in biological and social dynamics. A fundamental characteristic of these systems is " self-organization," where complex global patterns emerge from simple local interactions. While descriptive models like the Cucker-Smale model are well-established, a more challenging and practical problem lies in *optimal control* : How can an external agent (e.g., a sheepdog guiding sheep, or a traffic management system guiding autonomous vehicles) influence the group to achieve a specific goal with minimum energy?

The primary bottleneck in solving these problems is the "curse of dimensionality." In a system with N agents, calculating the interactions required to update the system's state typically scales with $O(N^2)$. For large-scale systems where N reaches thousands or millions, standard numerical optimization methods become prohibitively expensive and computationally infeasible.

This research project aims to develop, analyze, and implement a novel suite of high-efficiency numerical algorithms to solve these complex optimal control problems. We propose a hybrid approach that integrates classical optimization theory with modern stochastic approximation techniques. Specifically, we will

develop: 1. The Gradient-Random Batch Method (Gradient-RBM): Instead of calculating interactions between every pair of agents, this method utilizes small, randomly selected batches of agents to estimate the interaction forces. This reduces the computational complexity per time step from $O(N^2)$ to $O(NP)$, where P is a small batch size. 2. Proximal Alternating Block Coordinate Descent with RBM (PABCD-RBM): To further enhance efficiency, we propose an algorithm that updates the control strategies for only a subset (block) of agents at each iteration, rather than the entire population simultaneously.

Our preliminary results demonstrate that these methods can reduce computational time by orders of magnitude while maintaining high accuracy. The project will also deliver rigorous mathematical proofs of convergence for these stochastic algorithms.

All are welcome