Midterm Exam 1
Math 1013/1804 (2013-2014, Semester 2)
Practice Exam
Dr. Benjamin Kane

You have one hour and 50 minutes for the exam. There are 8 multiple choice questions (5 points each) and 2 long answer questions (10 points each). There are a total of 9 pages (this title page, 3 pages of multiple choice, 2 pages of long answer, and 3 pages for scratch paper). Please make sure that you have received all of the pages. Please write your name, student ID number, and tutorial group number on each sheet. If a page becomes detached from the other pages, then this will prevent it becoming lost. No calculators, cell phones, or other electronic devices will be allowed during the exam. Please shut these off at this time. For the multiple choice questions, you do not need to explain your answers. **For the multiple choice questions, it is possible that more than one of the answers is correct.** For example, the answer may be both **A AND C**. For the long answer questions, you must **explain your answers.** Final answers with no explanation on the long answer questions (questions 9 and 10) will receive **NO CREDIT!**

Please sign below to verify that you have read these rules. Good luck with the exam!

Last name (surname) First name (given name) ID number Group

Signature

Problem	Points
1	
2	
3	
4	
5	

Problem	Points
6	
7	
8	
9	
10	

Grade

Total (out of 60)

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Problem 1. (5 Points)

Suppose that f has an inverse and that $f(a) = b$, and $f(c) = d$. Which of the following must always be true?

A: If $d > b$, then $c > a$.
B: $f^{-1}(b) = a$
C: If $b = d$, then $a = c$.
D: If $a = d$, then $b = c$.

Answer:_____

Problem 2. (5 Points)

What is the limit

$$\lim_{x\to 0}\left(x\sin\left(\cos\left(\frac{1}{x}\right)\right)\right)?$$

A: 1
B: ∞
C: 0
D: Does not exist

Answer:_____

Problem 3. (5 Points)

Which of the following is

$$\frac{d}{dx}\left(\sqrt[3]{5x^2-2x}\left(\sqrt{3x^3-2x}\right)\right)?$$

A: $\frac{(10x-2)\left(9x^2-2\right)}{6(5x^2-2x)^{\frac{2}{3}}(3x^3-2x)^{\frac{1}{2}}}$
B: $\frac{195x^{\frac{17}{6}}-66x^{\frac{11}{6}}-70x^{\frac{5}{6}}+20x^{-\frac{1}{6}}}{6\sqrt[3]{(5x-2)^2}\left(\sqrt{3x^2-2}\right)}$
C: $\frac{195x^4-66x^3+70^2+20x}{6\sqrt[3]{(5x^2-2x)^2}\left(\sqrt{3x^3-2x}\right)}$
D: $\frac{\sqrt{3x^3-2x}(10x-2)}{3(5x^2-2x)^{\frac{2}{3}}}+\frac{\sqrt[3]{5x^2-2x}\left(9x^2-2\right)}{2(3x^3-2x)^{\frac{1}{2}}}$

Answer:_____

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Problem 4. (5 Points)

Which of the following is/are (always) true?

A: If a function f is continuous on an interval (a, b) and $f(a) < y < f(b)$, then there exists $c \in (a, b)$ for which $f(c) = y$.

B: If a function f is differentiable on $[a, b]$, then there exists $c \in (a, b)$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

C: If a function is not continuous on an interval $[a, b]$ and $f(a) < y < f(b)$, then there is no $c \in (a, b)$ for which $f(c) = y$.

D: If a function is differentiable on $[a, b]$ and $f(a) < y < f(b)$, then there exists $c \in (a, b)$ for which $f(c) = y$.

Answer:_____

Problem 5. (5 Points)

What is

$$\lim_{x\to\infty} \frac{\sin(x)}{x}?$$

A: ∞ B: 1
C: 0 D: Does not exist.

Answer:_____

Problem 6. (5 Points)

Which of the following statements is/are true?

A Every horizontal line hits the graph of a function at most once.
B Every vertical line hits the graph of a function exactly once.
C Every vertical line hits the graph of an injective function at most once.
D Every horizontal line hits the graph of an injective function at most once.

Answer:_____

Problem 7. (5 Points)

Suppose that

$$f(x) := \frac{x^2 - 1}{x^2 + 2rx - 4}$$

For which of the following values of $r \in \mathbb{R}$ is

$$f'(0) = 1?$$

A: -2 B: 8
C: -8 D: None of the above

Answer:_____

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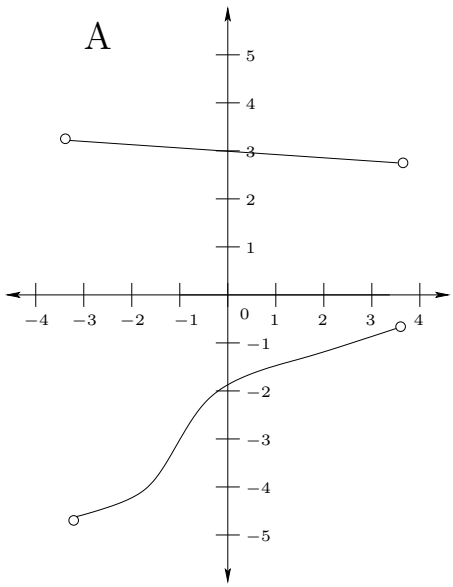
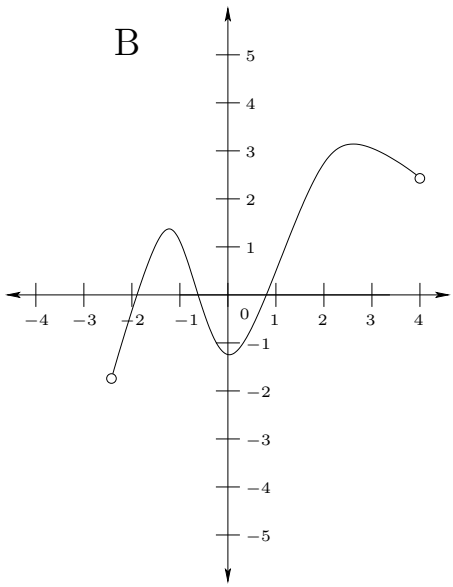
Problem 8. (5 Points)

Which of the following graphs is the graph of a function which has an inverse? The entire graph is what you can see (lines do NOT continue forever, for example)!

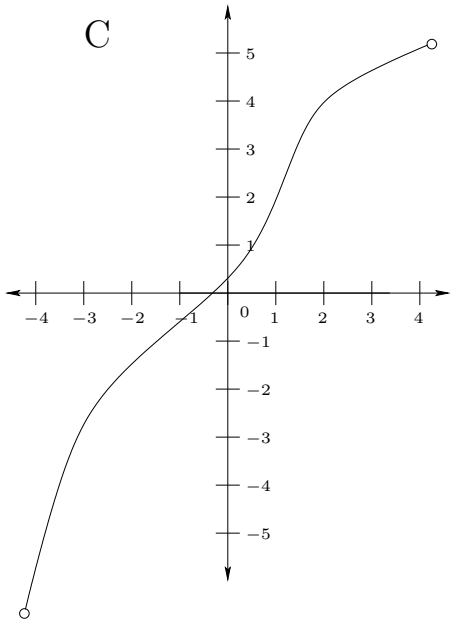
A



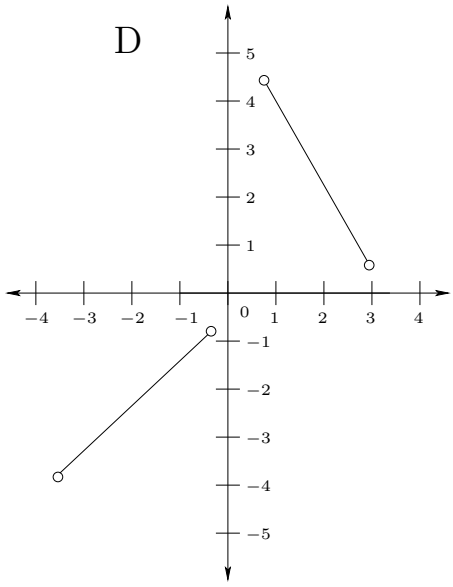
B



C



D



Answer:_____

Name, ID Number, Group: _____

Problem 9. (10 Points)

Suppose that y is a function of x which satisfies

$$x^2 \cos(2\pi y) + y \sin\left(\frac{\pi x}{2}\right) = 1.$$

(a) Solve for $\frac{dy}{dx}$ as a function of x and y (that is to say, you should give a function so that if someone gives you x and y, you can tell them $\frac{dy}{dx}$).

(b) Find $\frac{dy}{dx}$ at the point where $x = 1$ and $y = 0$.

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Problem 10. (10 Points)
Show that there exists at least one $x \in \mathbb{R}$ for which

$$x\cos(x) = 1.$$

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Scratch Paper

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Scratch Paper

Name, ID Number, Group: _____

Scratch Paper