香港大學數學系主辦公開講座

數趣漫話

之

運籌學漫遊

圖與網絡

S.C.K. Chu

朱進強

2006.11.11

A Graph and Network Excursion in Operations Research

「理論—啟發—算法—啟發—應用—啟發—理論」

Theory -- Algorithm -- Application -- Theory

GRAPH	樹 Tree	歐拉圈 Euler cycle	匹配 Matching	哈密頓圈 Hamilton cycle
NETWORK	MST	CPP	AP	TSP

圖與網絡

TSP: Travelling Salesman Problem
(行進)推銷員問題

AP: Assignment Problem
分配問題

CPP: Chinese Postman Problem
中國郵遞員問題

MST: Minimum Spanning Tree
最優(小)支撐樹

運籌學

「理論—啟發—算法—啟發—應用—啟發—理論」

Theory -- Algorithm -- Application -- Theory

GRAPH	1 支撐樹 Spanning Tree	歐拉圈 Euler cycle	匹配 Matching	哈密頓圈 Hamilton cycle
NETWORK	MST	CPP	AP	TSP

圖與網絡

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1
MST: Minimum Spanning Tree
最優(小)支撐樹

運籌學

「理論—啟發—算法—啟發—應用—啟發—理論」

Theory -- Algorithm -- Application -- Theory

GRAPH	支撐樹 Spanning Tree	2 歐拉圈 Euler cycle	匹配 Matching	哈密頓圈 Hamilton cycle
NETWORK	MST	CPP	AP	TSP

圖與網絡

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運籌學

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Theory -- Algorithm -- Application -- Theory

GRAPH	支撐樹 Spanning Tree	歐拉圈 Euler cycle	3 匹配 Matching	哈密頓圈 Hamilton cycle
NETWORK	MST	CPP	AP	TSP

圖與網絡

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運籌學

「理論—啟發—算法—啟發—應用—啟發—理論」

Theory -- Algorithm -- Application -- Theory

GRAPH	支撐樹 Spanning Tree	歐拉圈 Euler cycle	匹配 Matching	4 哈密頓圈 Hamilton cycle
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圖與網絡

TSP: Travelling Salesman Problem
(行進)推銷員問題 4

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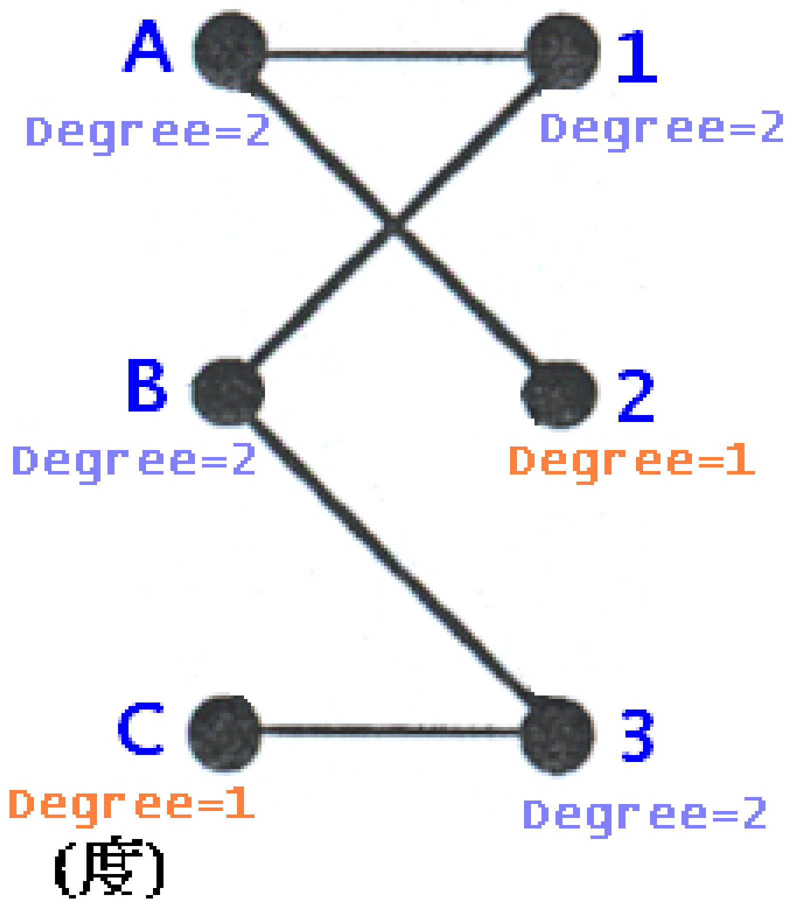
中國郵遞員問題

MST: Minimum Spanning Tree
最優(小)支撐樹

運籌學

圖 Graph G

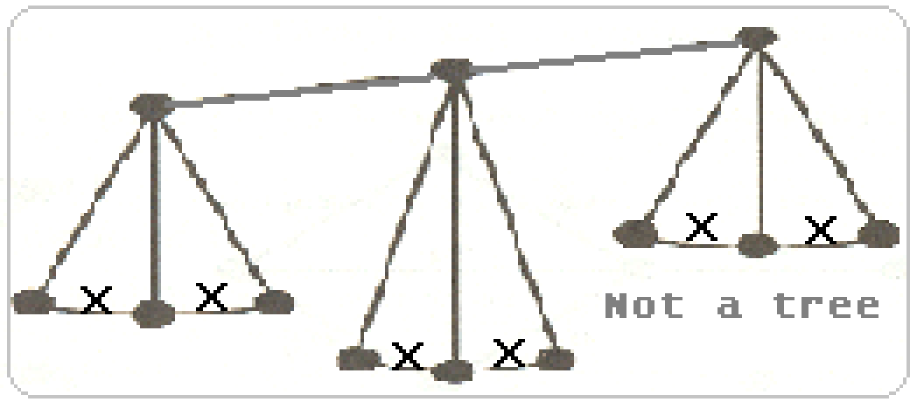


點: Point, Vertex, Node

V={A,B,C,1,2,3} |V|=6

邊: Line, Edge, Arc

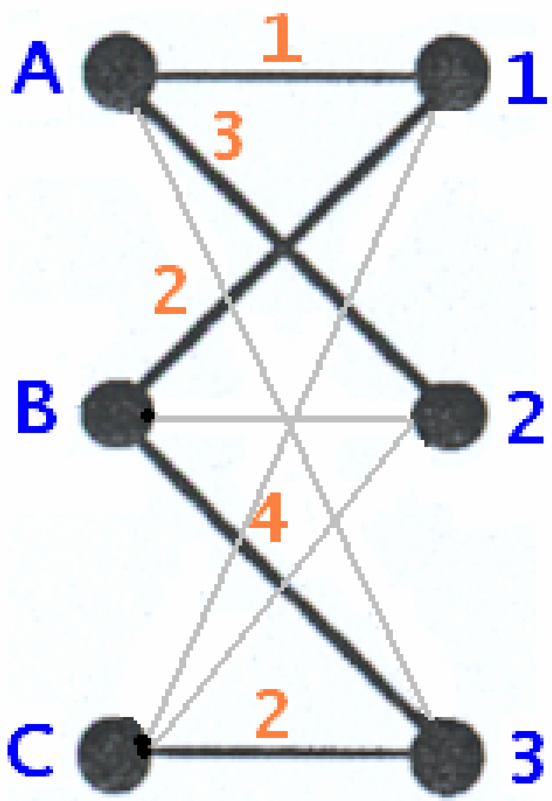
E={(A,1),(A,2),(B,1),
(B,3),(C,3)} |E|=5



(連通) (圈) (支撐樹)

connected + no cycle => SPANNING TREE

網絡Network G=(V,E)



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V={A,B,C,1,2,3} |V|=6

邊: Line, Edge, Arc

E={(A,1),(A,2),(B,1),
(B,3),(C,3)} |E|=5

價: Cost, Weight

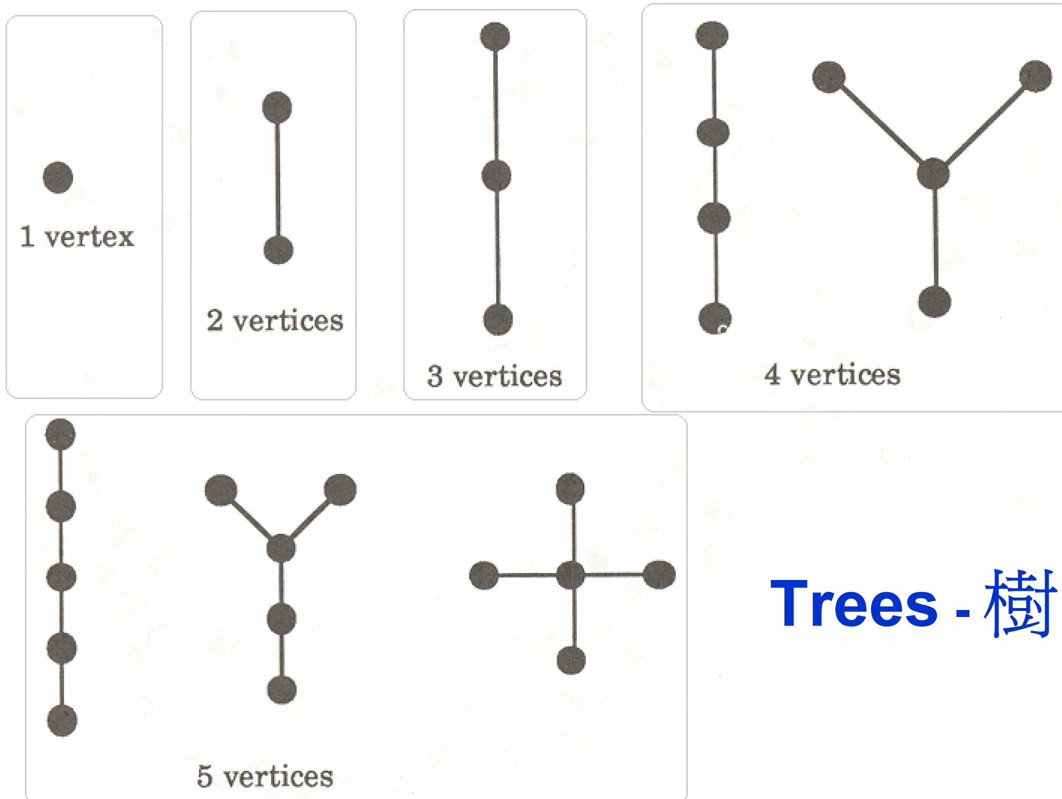
c(A,1)=1,...,c(C,3)=2

(支撐樹之造價)

Cost of SPANNING TREE = 1+3+2+4+2 = 12

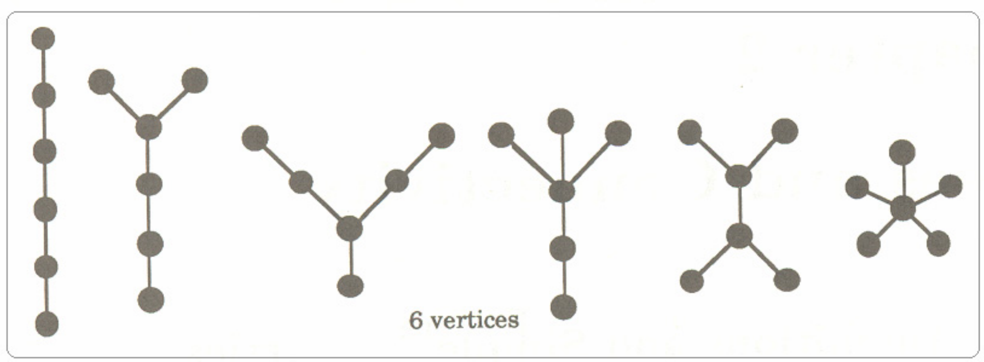
Trees - 樹圖

4 edges



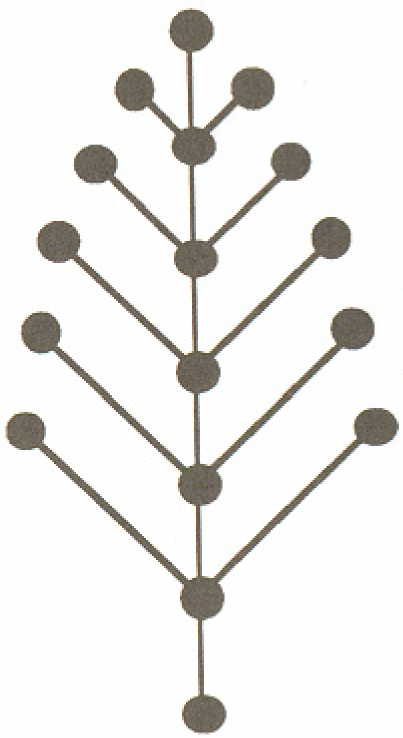
The trees with at most five vertices.

5 edges

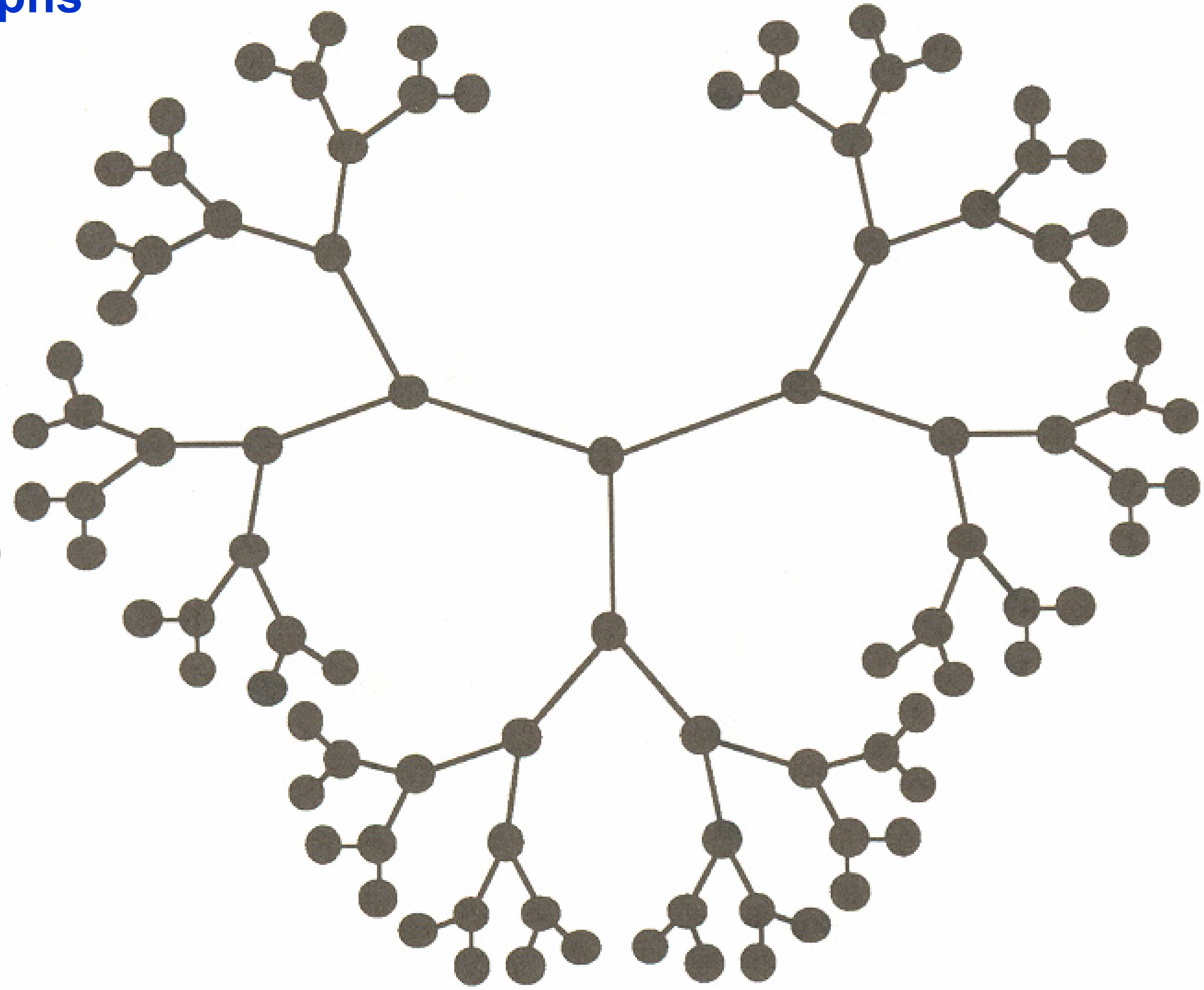


The trees with six vertices.

樹圖 Tree Graphs



17 vertices
16 edges

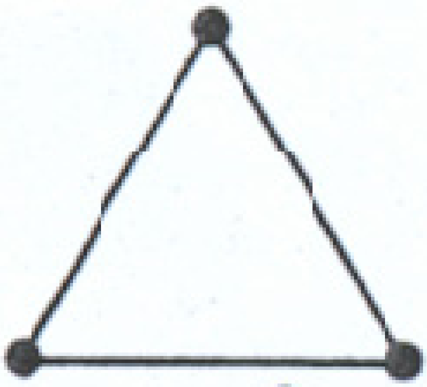


94 vertices
93 edges

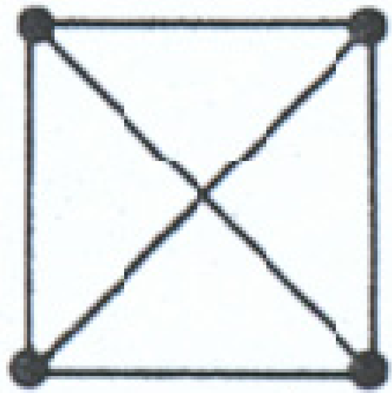
Two fancier trees.

 # edges = n(n-1)/2

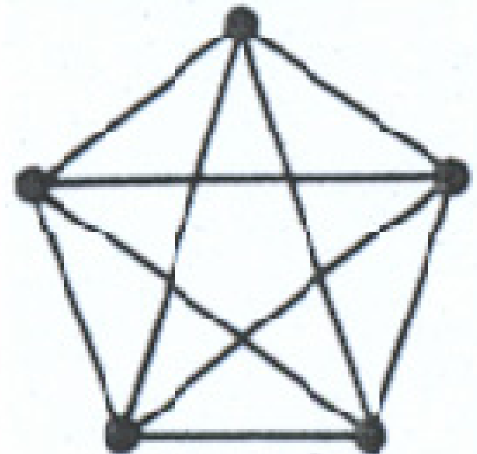
Complete Graph -each pair of vertices connected
(完全圖)



$n = 3$

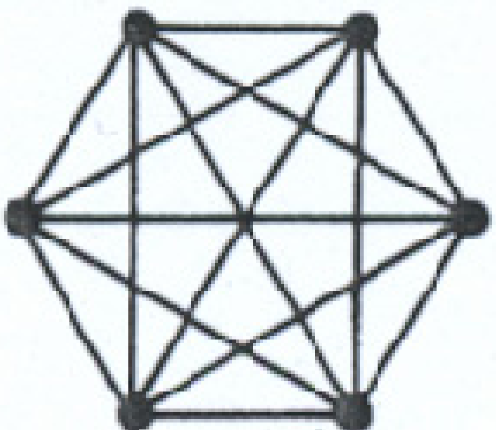


$n = 4$

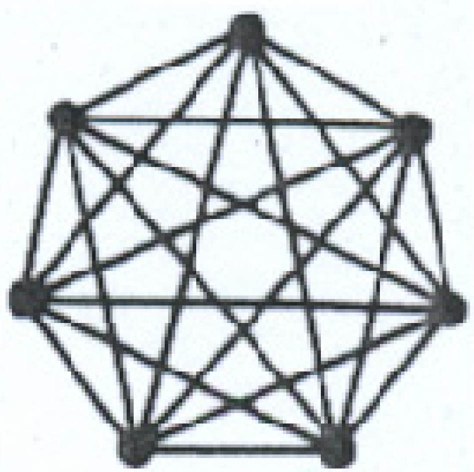


$n = 5$

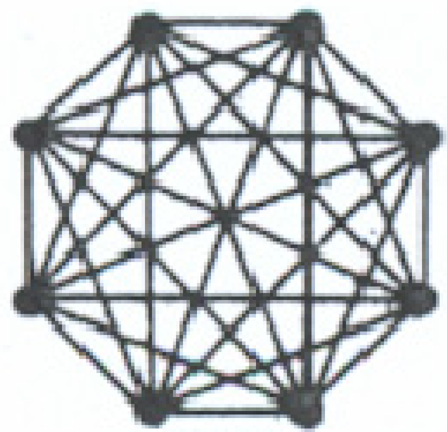
 # edges = 15



$n = 6$



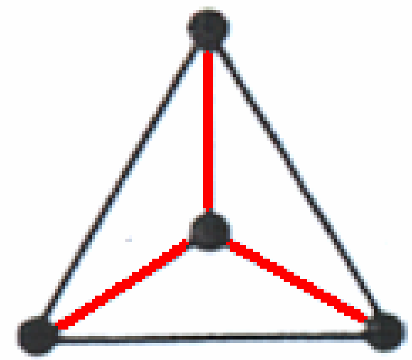
$n = 7$



$n = 8$

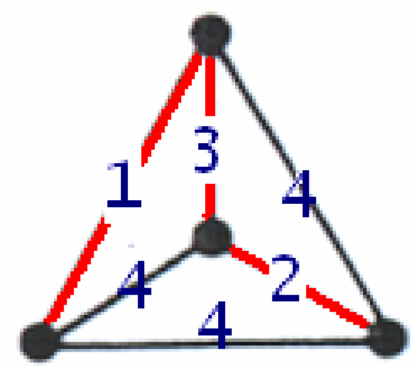
Graph
圖

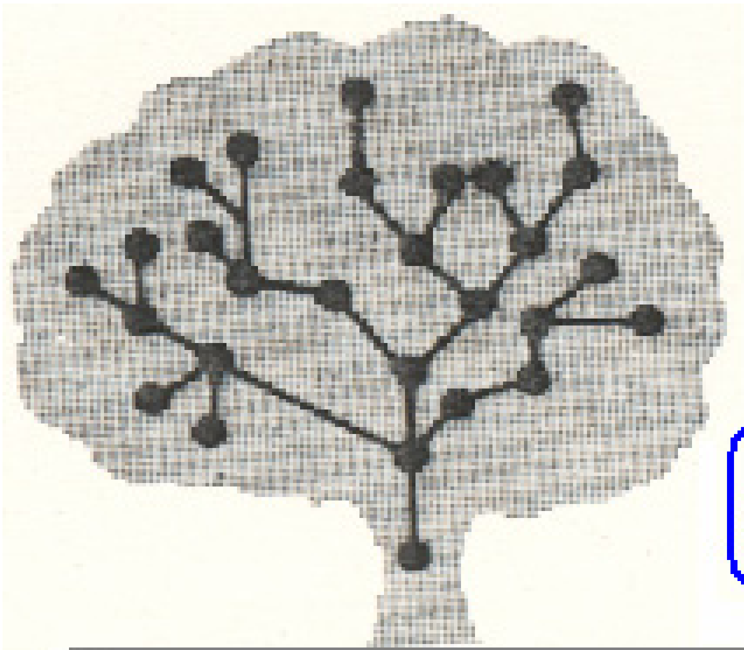
Tree
支撐樹



Network
網絡

Minimum Spanning Tree(MST)
最優支撐樹





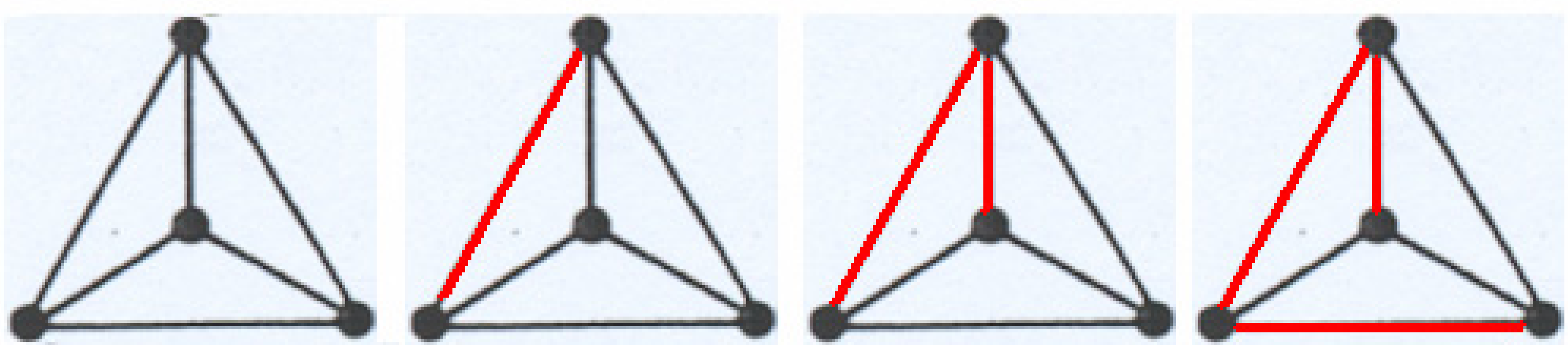
(支撐樹)

Spanning Tree & Minimum Spanning Tree

短視之啓發式方法

Algorithm:
Myopic Heuristic works!

Theory:
Spanning Tree <=> Any 2 of the conditions:
{ no cycle , connected , |E| = |V|-1 }

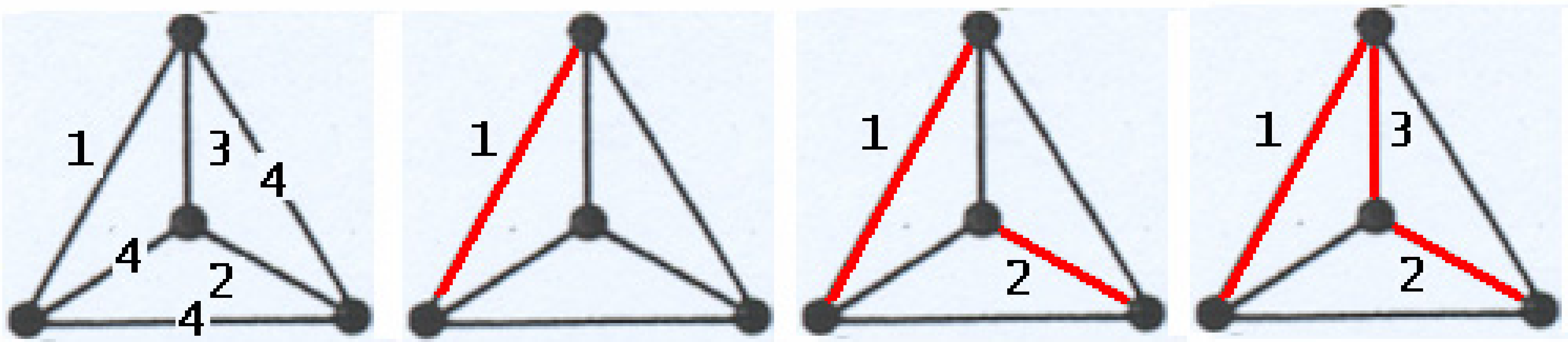


Application: Minimum Spanning Tree 最優支撐樹

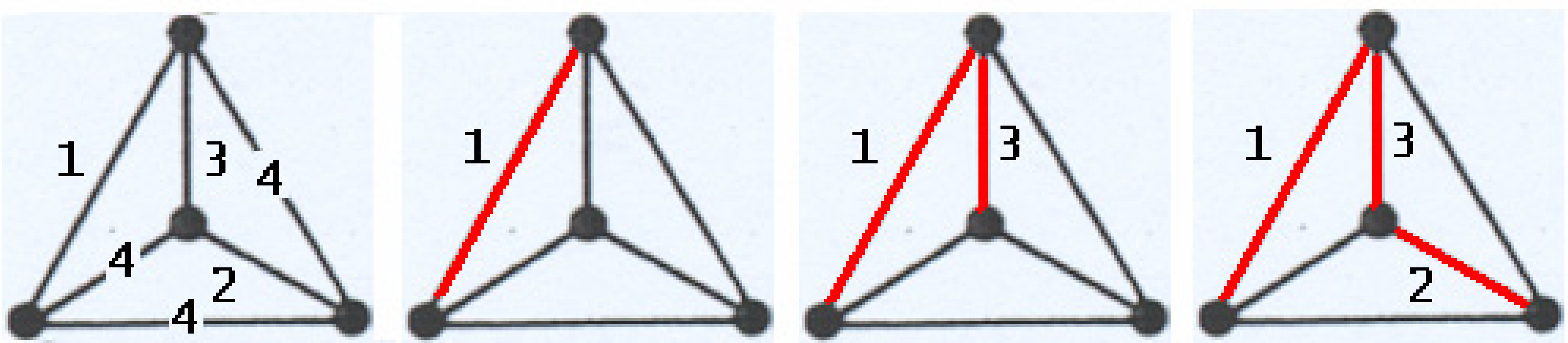
（貪婪之啓發式方法）

Algorithm:
Greedy Heuristic works!

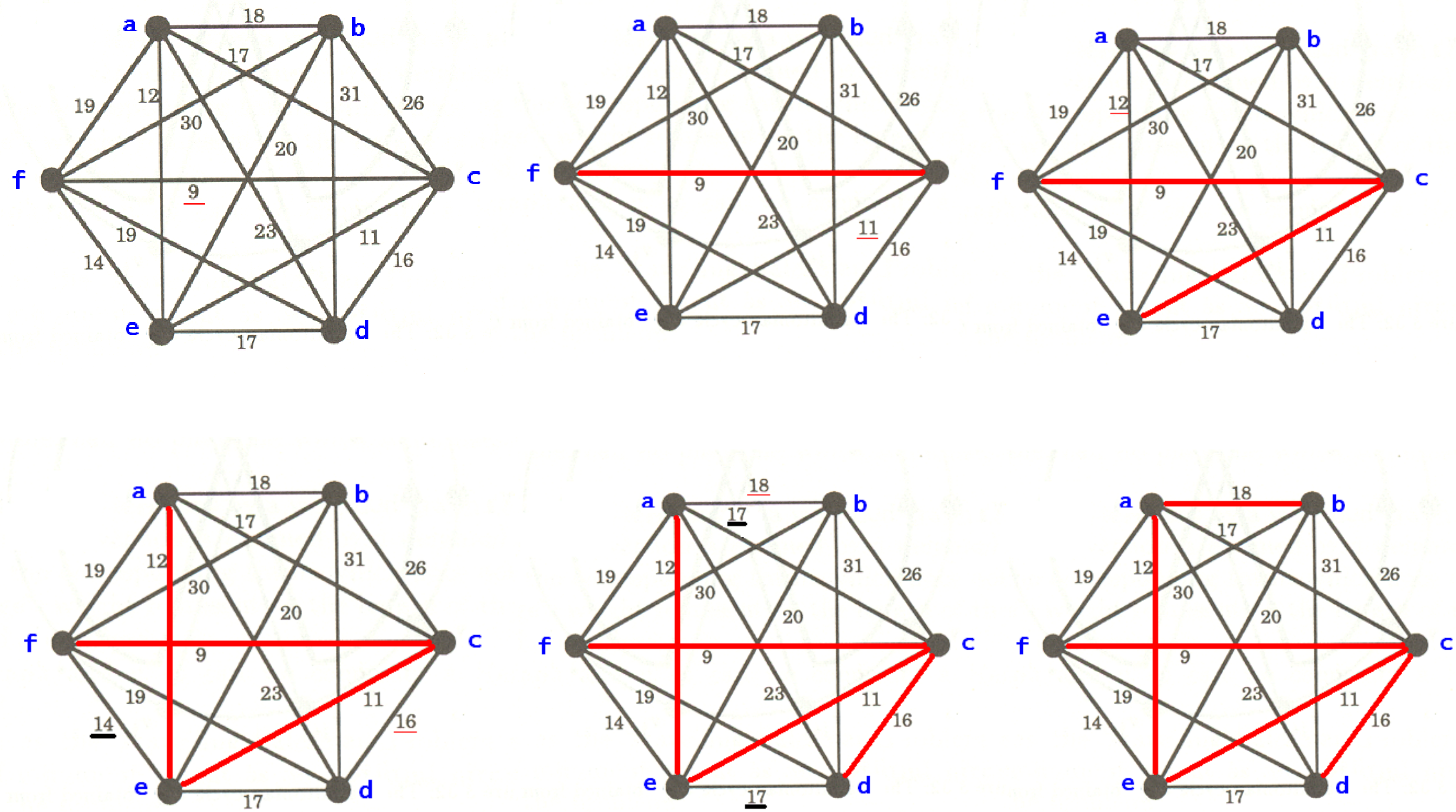
Kruskal's method



Prim's method



[NOTE: # different spanning trees = n^(n-2) = 6^4 = 1296 *(Cayley, 1889)*]



最優(最小)支撐樹
(Cost = 66)

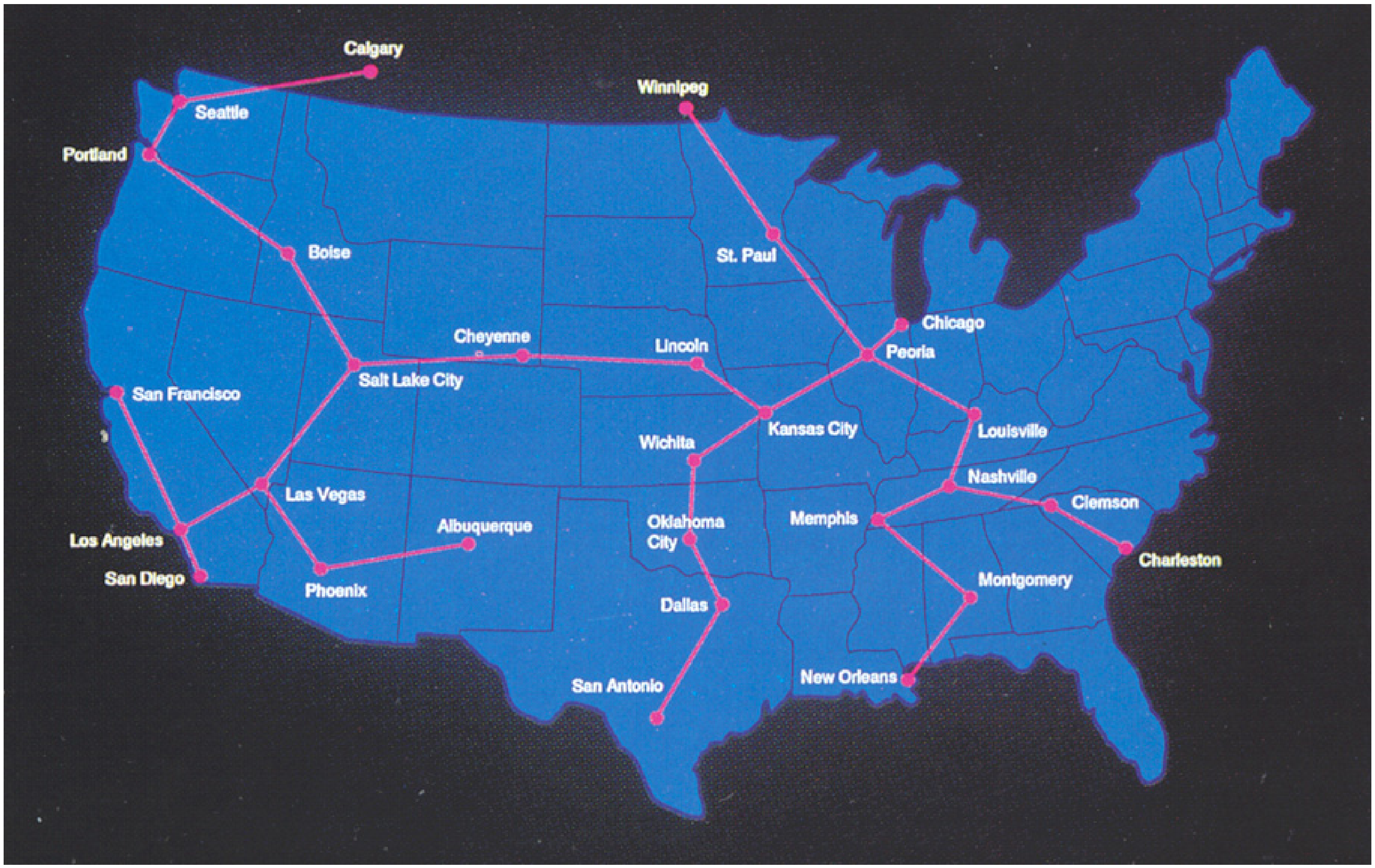
9, 11, 12, 14, 16, 17, 17, 18, 19, 19, 20, 23, 26, 30, 31

A bigger example on Minimum Spanning Tree construction.

An Illustration of Minimum Spanning Tree

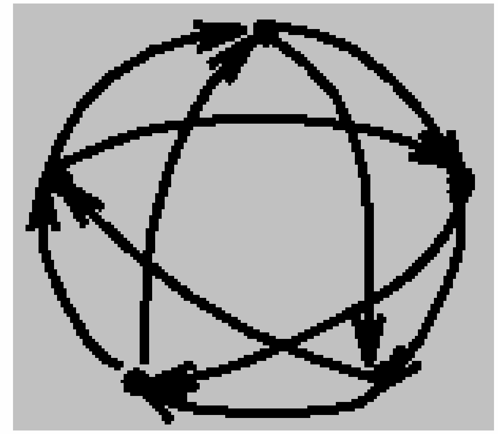
Communication Network Application [7600 miles for 29 cities]

(source: Tannenbaum & Arnold)



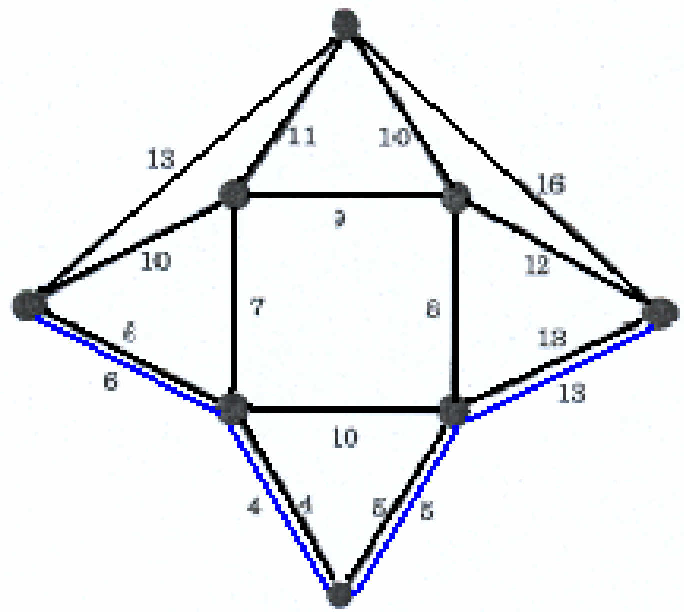
Graph
圖

Euler
Cycle
歐拉圈



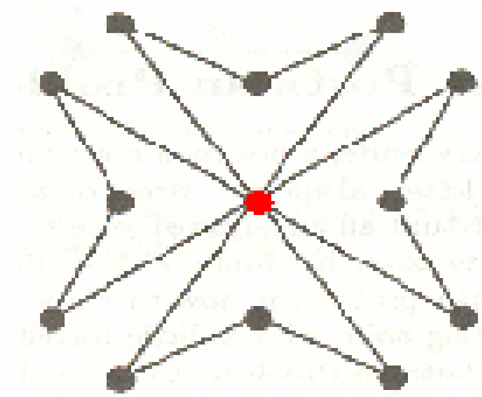
Network
網絡

Chinese
Postman
Problem
中國郵遞員問題

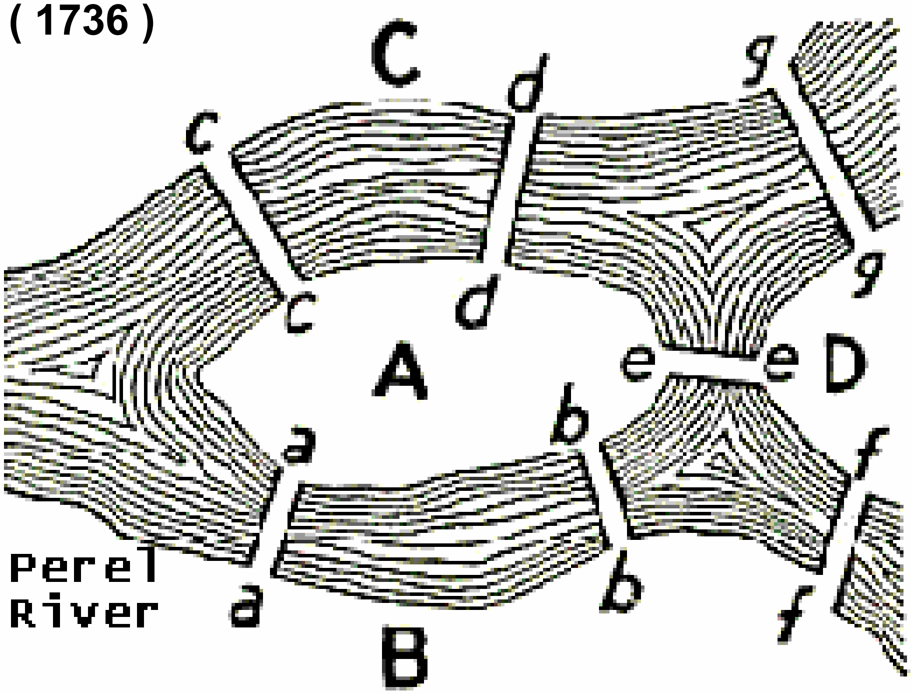


(歐拉圈)

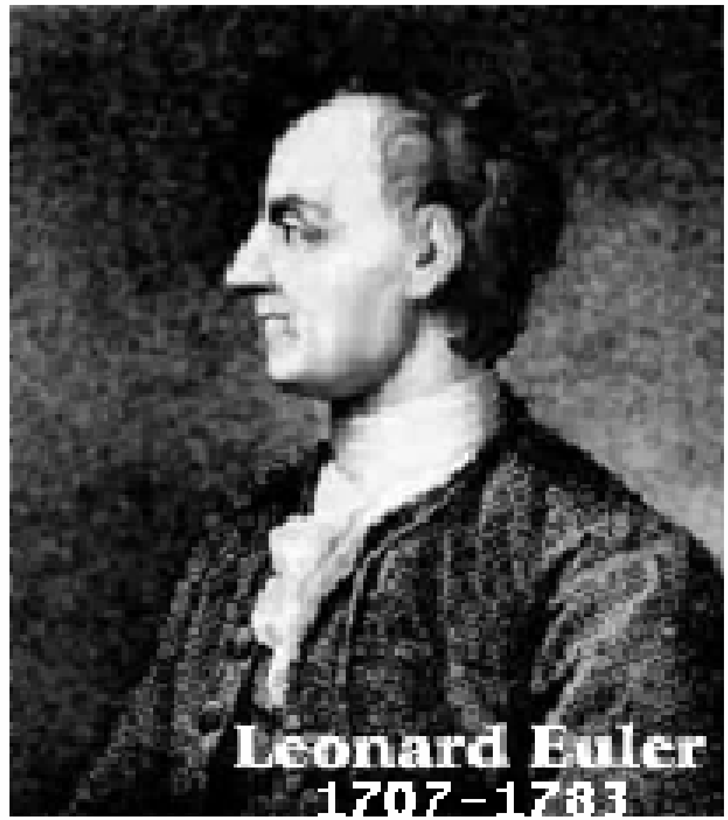
Euler Cycle & Chinese Postman Problem (CPP)



(1736)



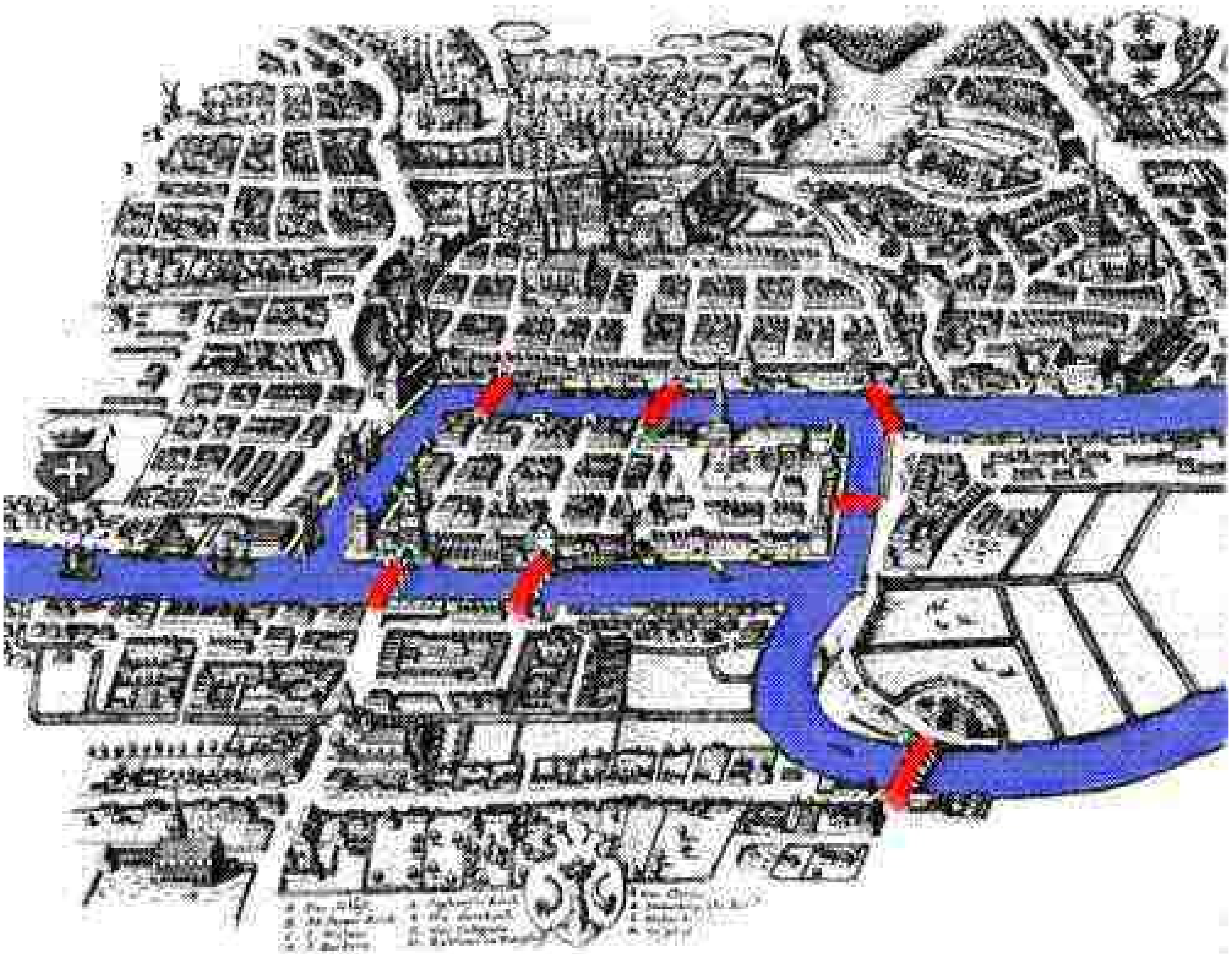
Geographic Map:
The Königsberg Bridges.

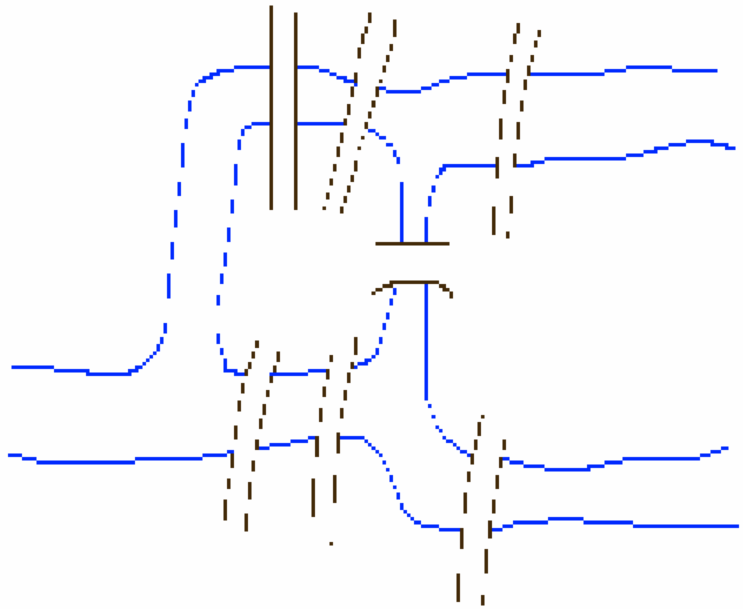


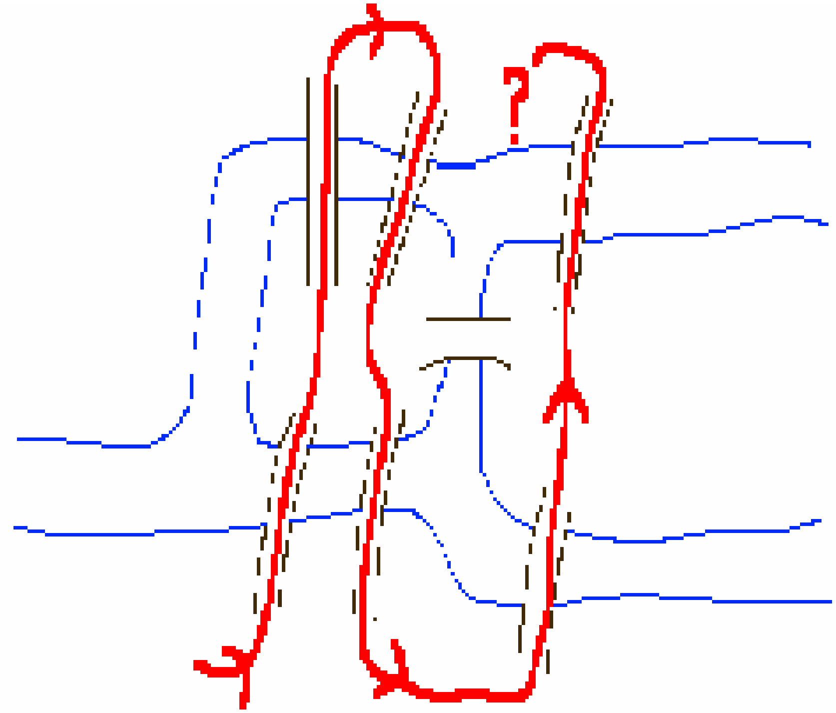
The Konigsberg's seven bridges problem.

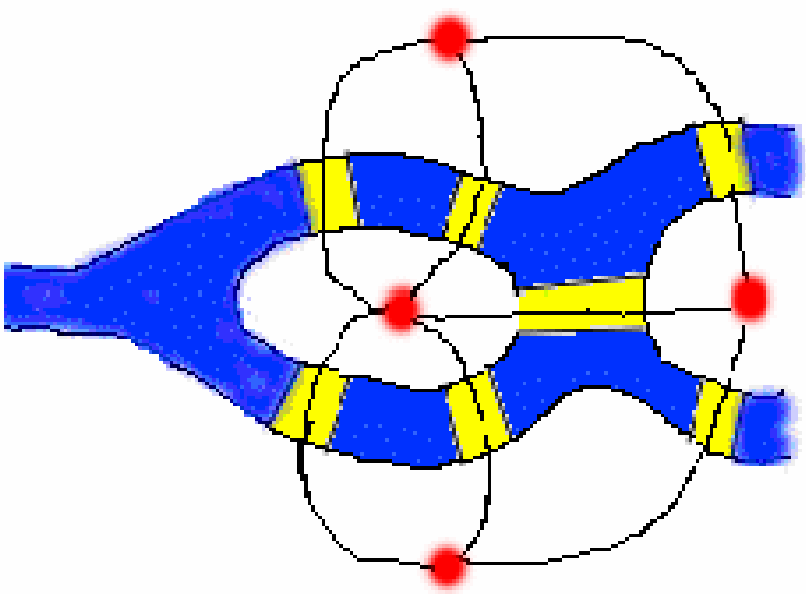
City of Konigsberg with 7 bridges over Perel River

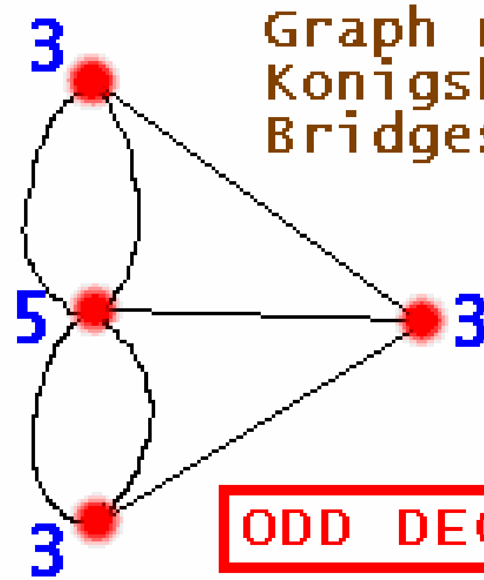
City of Konigsberg with 7 bridges over Perel River









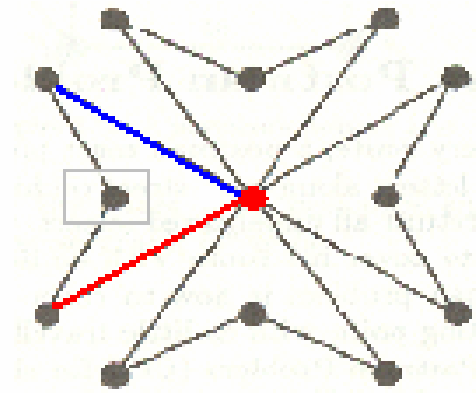


Graph model of Konigsberg Bridges

ODD DEGREES!!

(歐拉圈)

Euler Cycle & Chinese Postman Problem (CPP)

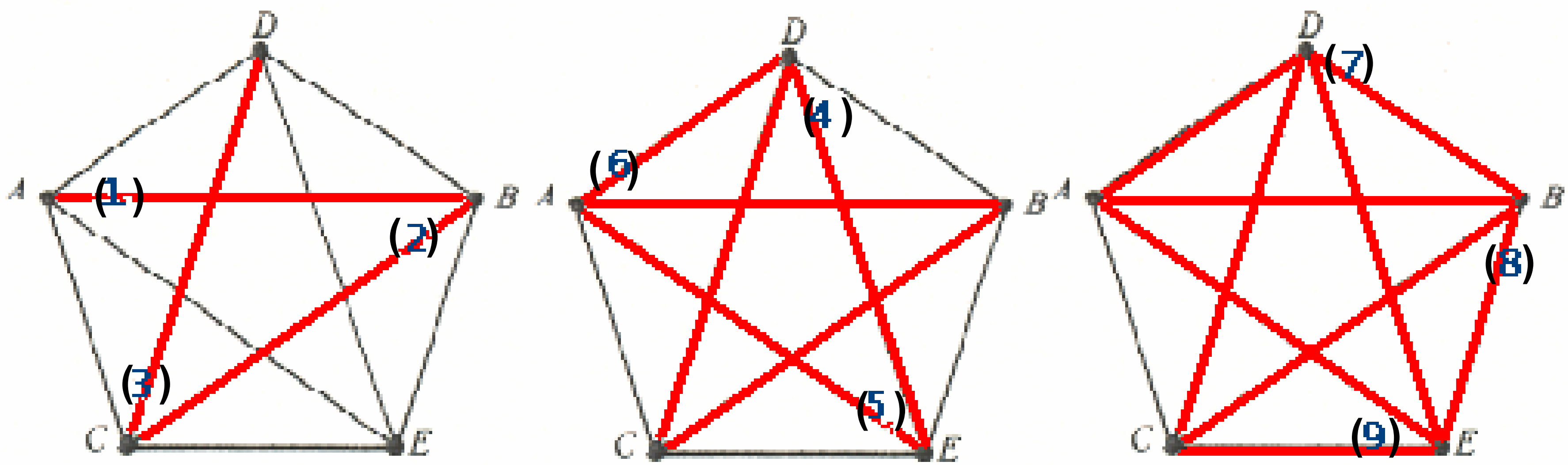


Theorem

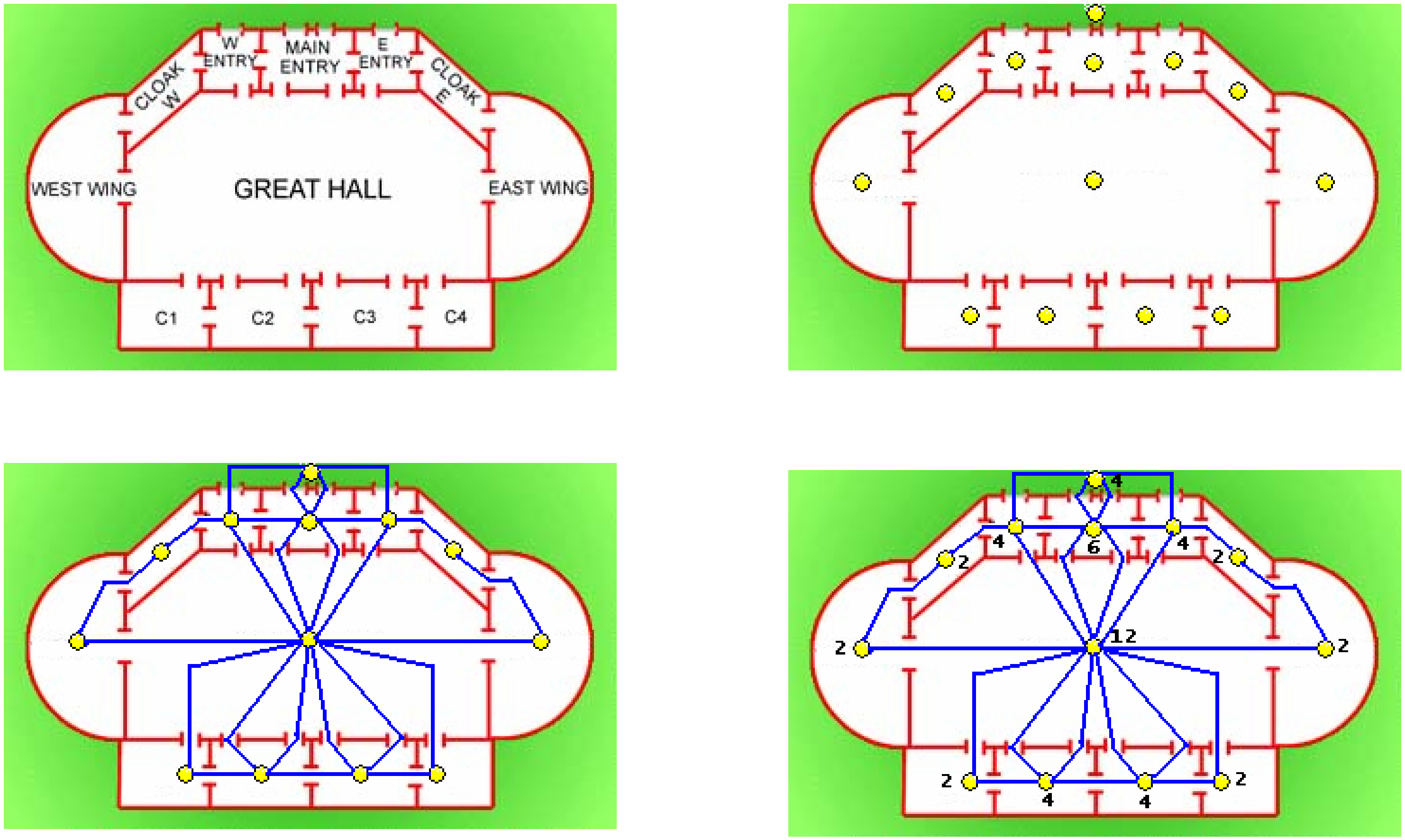
A connected graph is Euler if and only if the degree of every vertex is even.

Algorithm

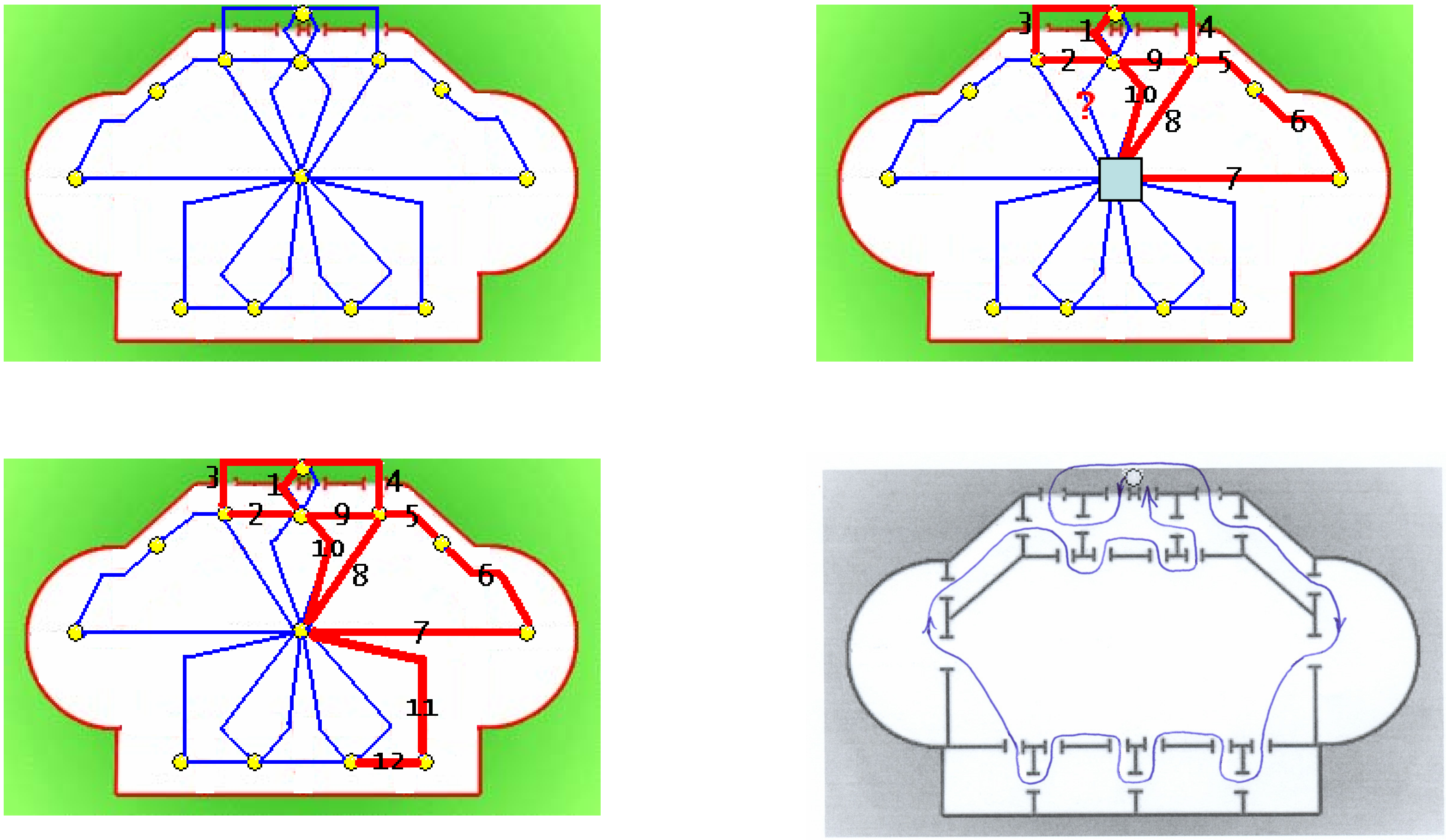
Fleury's Heuristic with no-bridge rule.



An illustration of Euler cycle modelling application: The Great Hall Tour of 25 Doors



An illustration of Euler cycle modelling application: The Great Hall Tour of 25 Doors

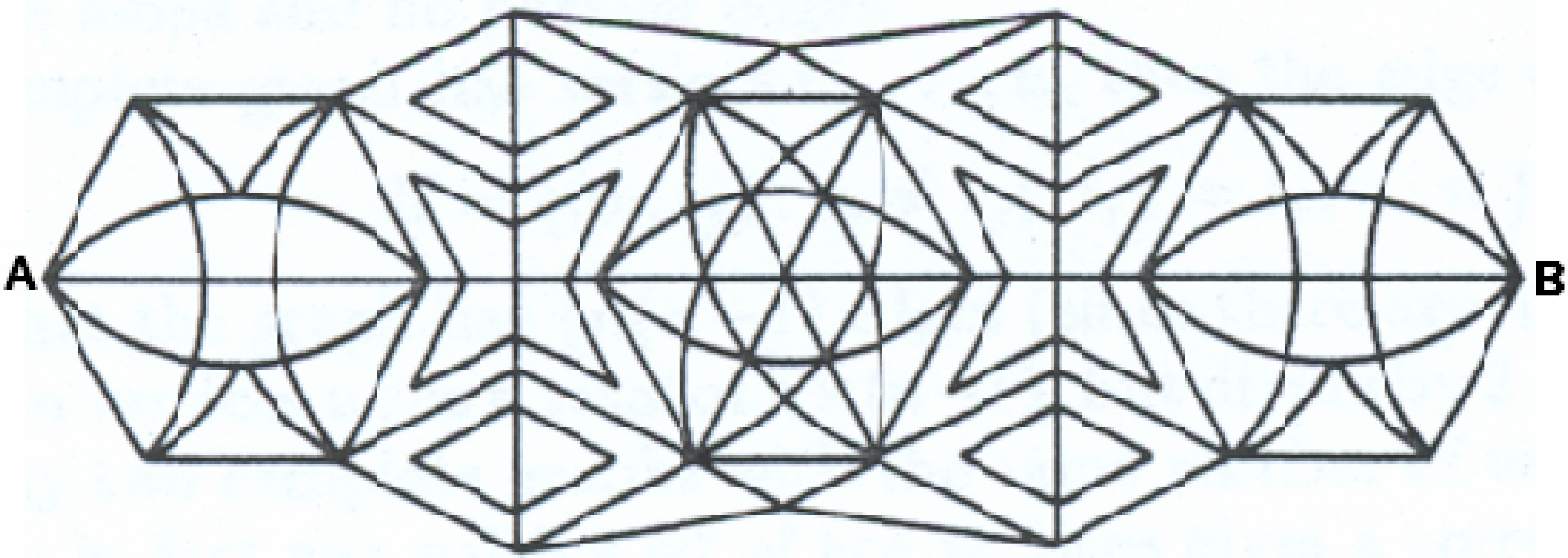


Game of "Dragon-Tracing Puzzle" (一筆劃)

Theory.
Euler Path - exactly 2 vertices of odd degrees.

Algorithm.
Fleury's method.

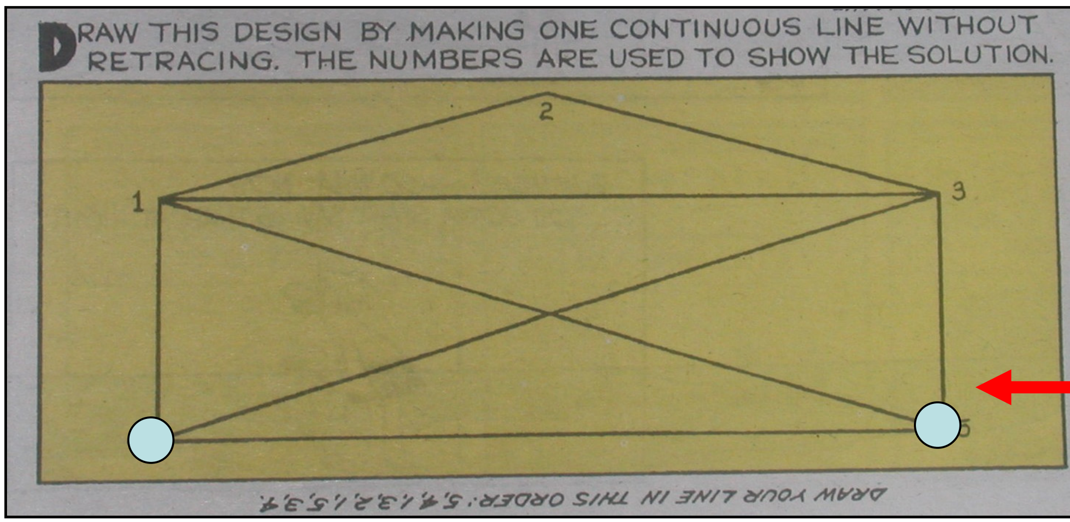
EulerCycleCode

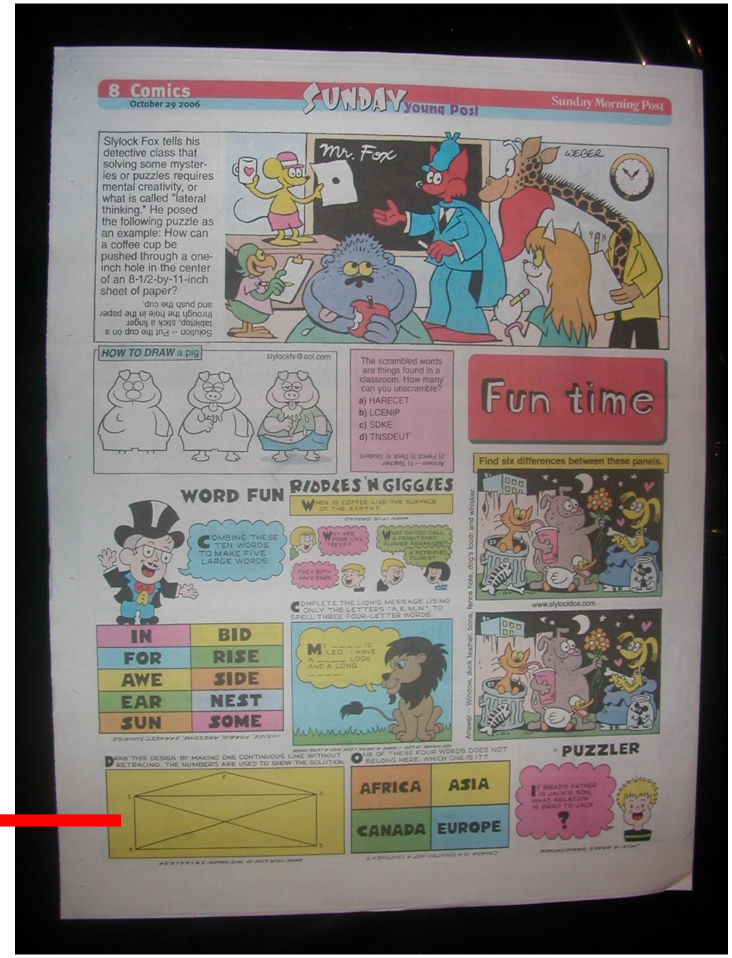


[Degree of A and B = 5 (odd); all others even]

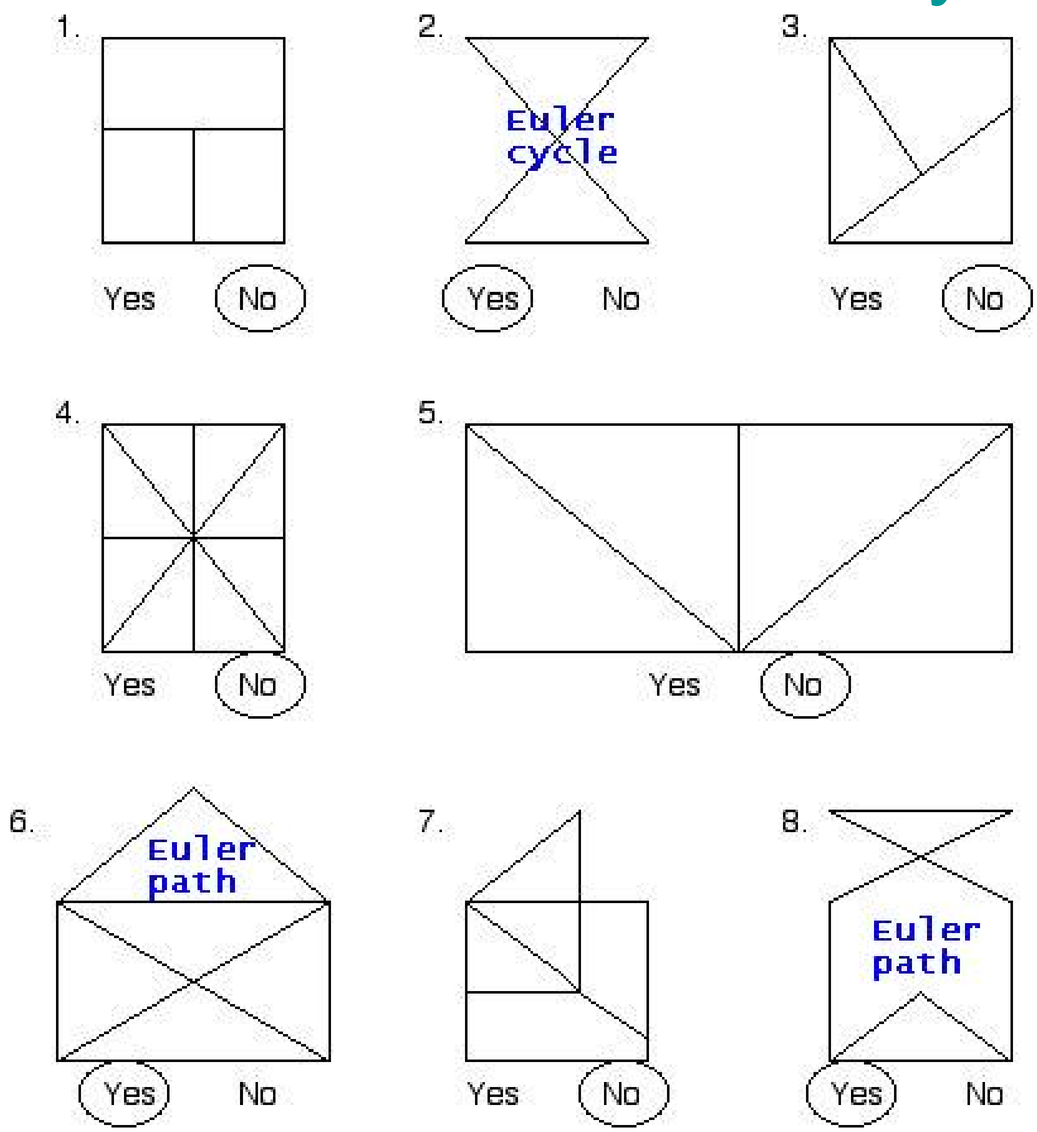
SCMP - Sunday Morning Post

October 29 2006

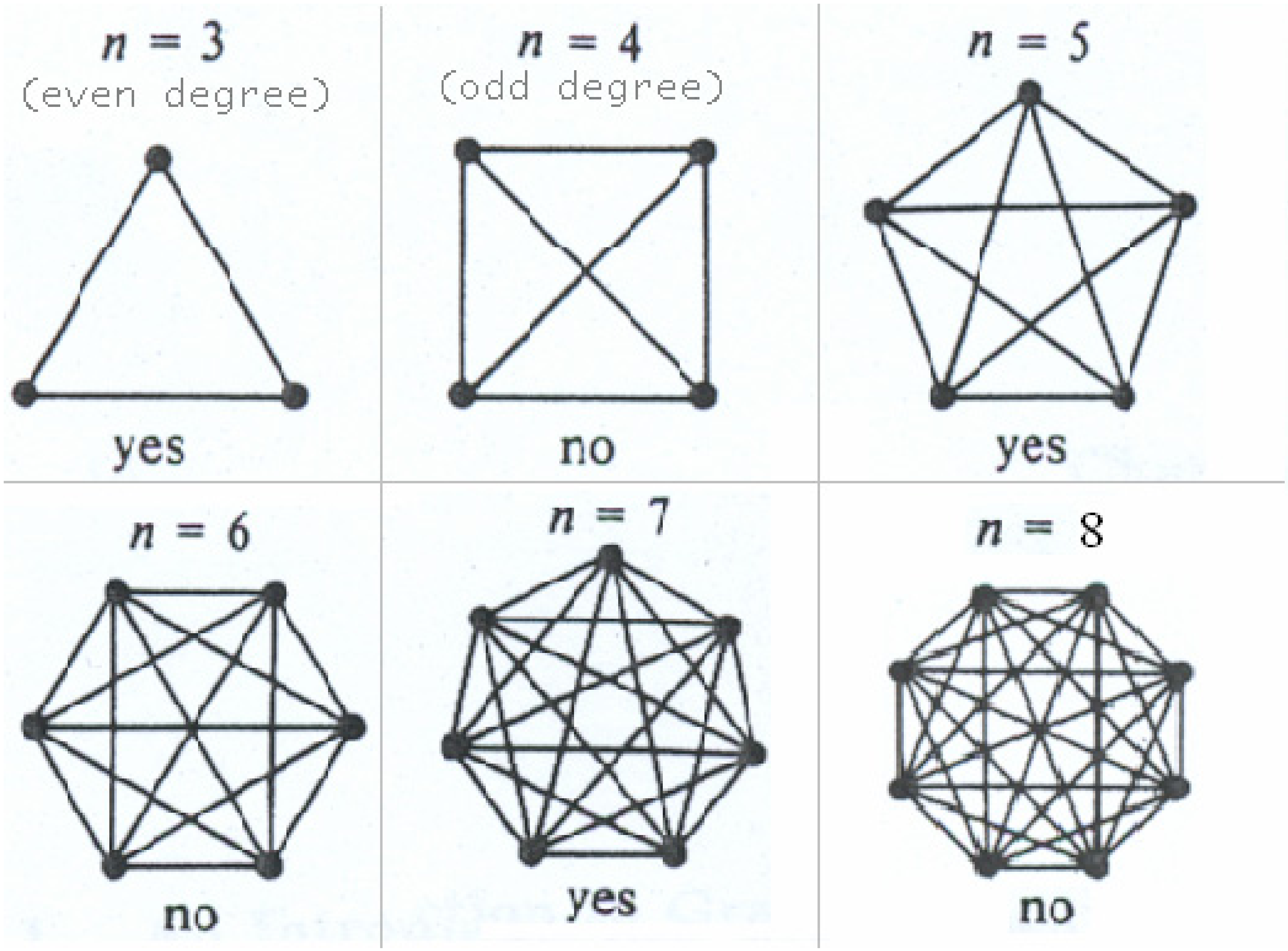




Examples on existence of Euler cycles/paths



Examples on Euler cycles on complete graphs



Euler Cycle & Chinese Postman Problem

（中國郵遞員問題）

Application

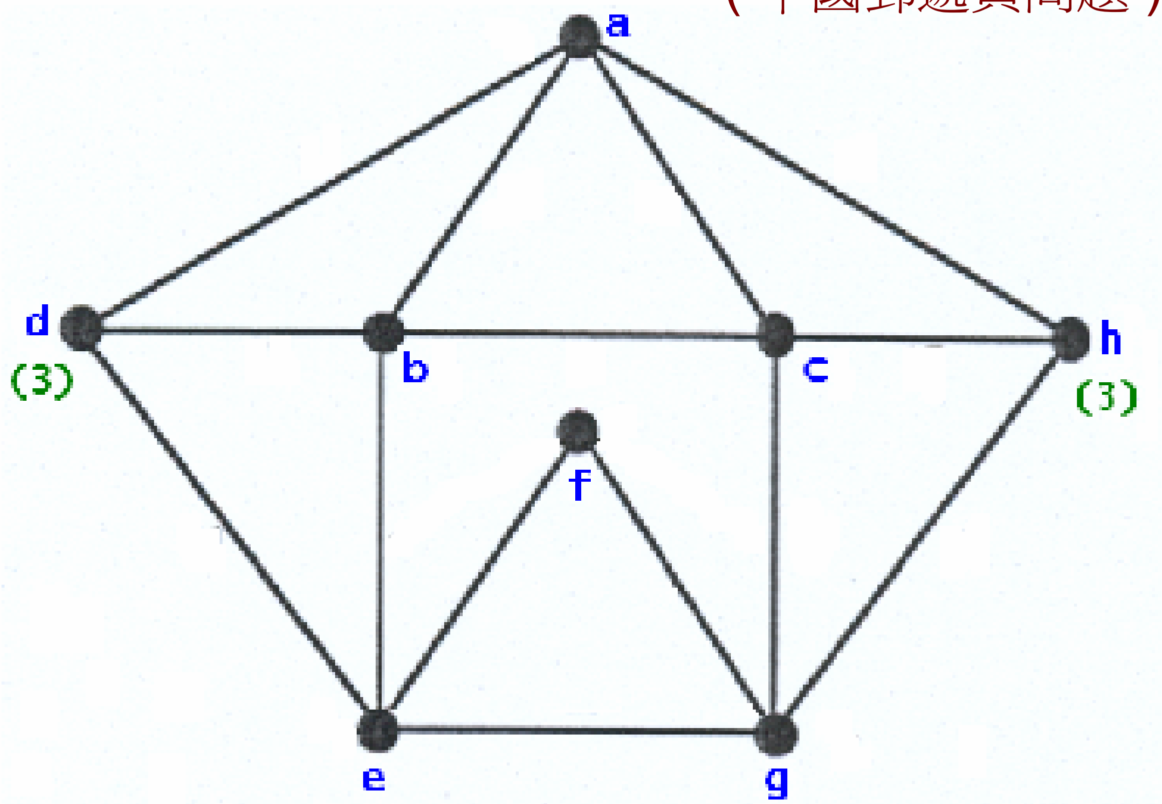
Algorithm:
Euler cycle plus shortest paths.

一個郵遞員，每次送信，要走遍他負責的投遞範圍內的每條街道，完成送信任務後回到郵局，他應按什麼樣的路線走，使所走的總路程最短呢?

郵遞員最優投遞路線的問題是由我國的管梅谷教授首先 (1962) 提出並研究的，國際上現在稱之爲中國郵遞員問題(Chinese Postman Problem).

Euler Cycle & Chinese Postman Problem

(中國郵遞員問題)

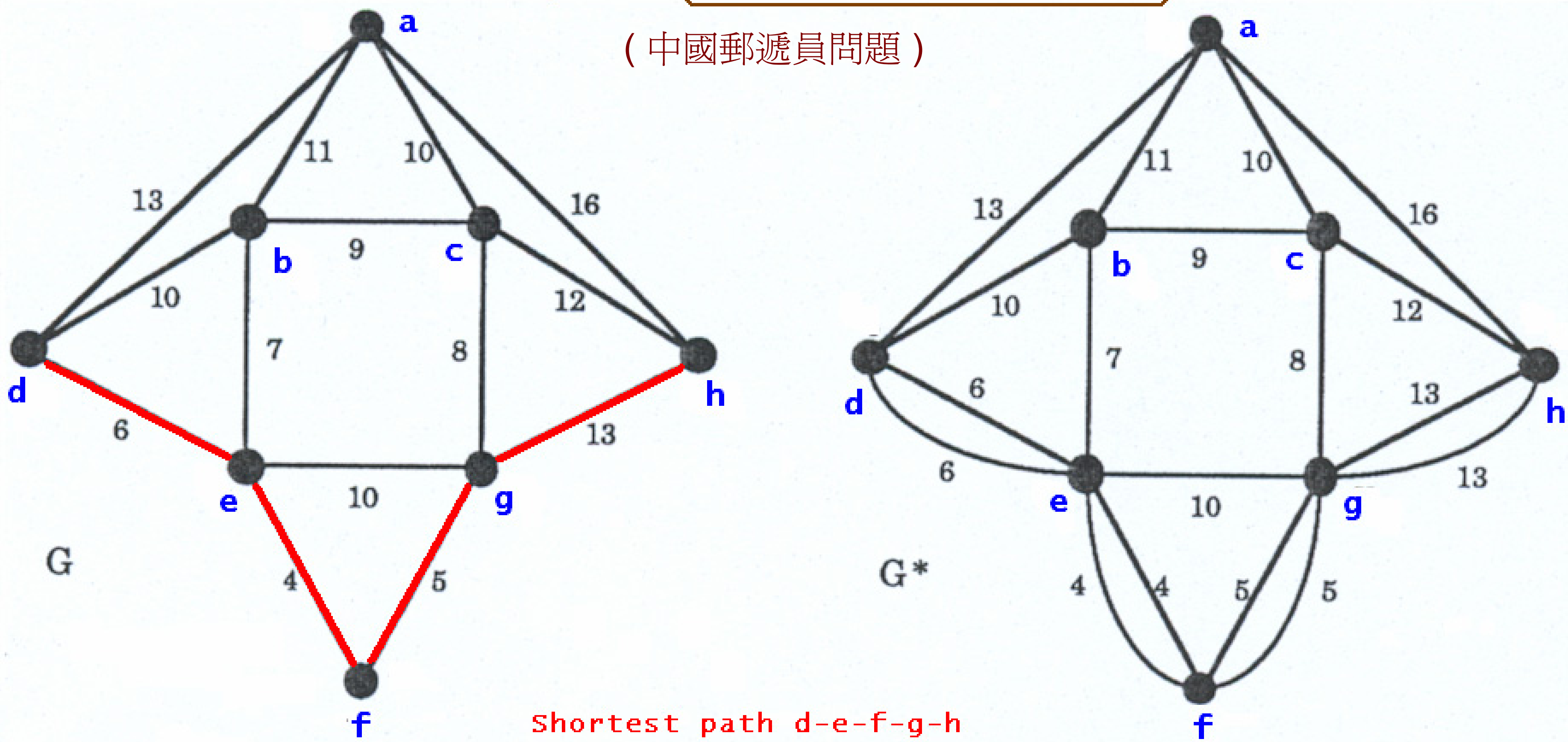


One possible solution:

(1) Euler path d-a-b-d-e-f-g-e-b-c-a-h-c-g-h
plus
(2) Shortest path h-g-f-e-d (back from h to d)

Euler Cycle & Chinese Postman Problem

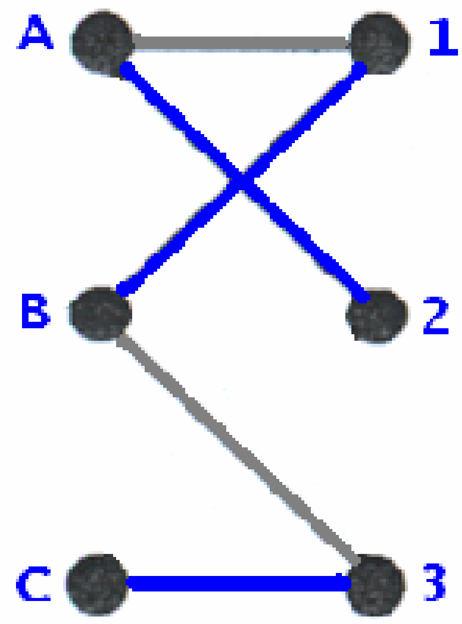
（中國郵遞員問題）



A graph G with an Euler supergraph G^* obtained by duplicating edges.

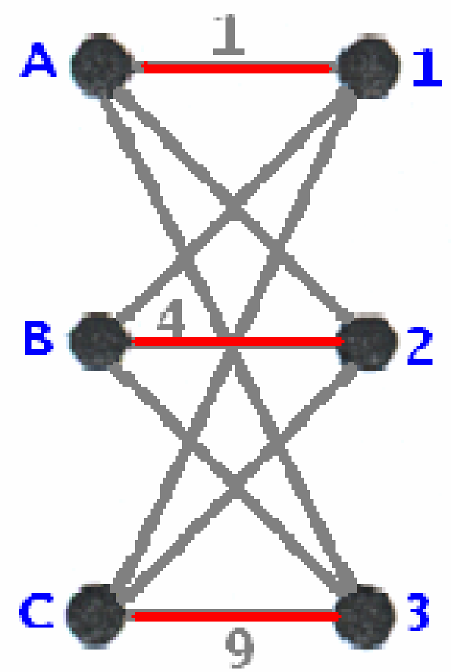
Graph
圖

Matching
匹配



Network
網絡

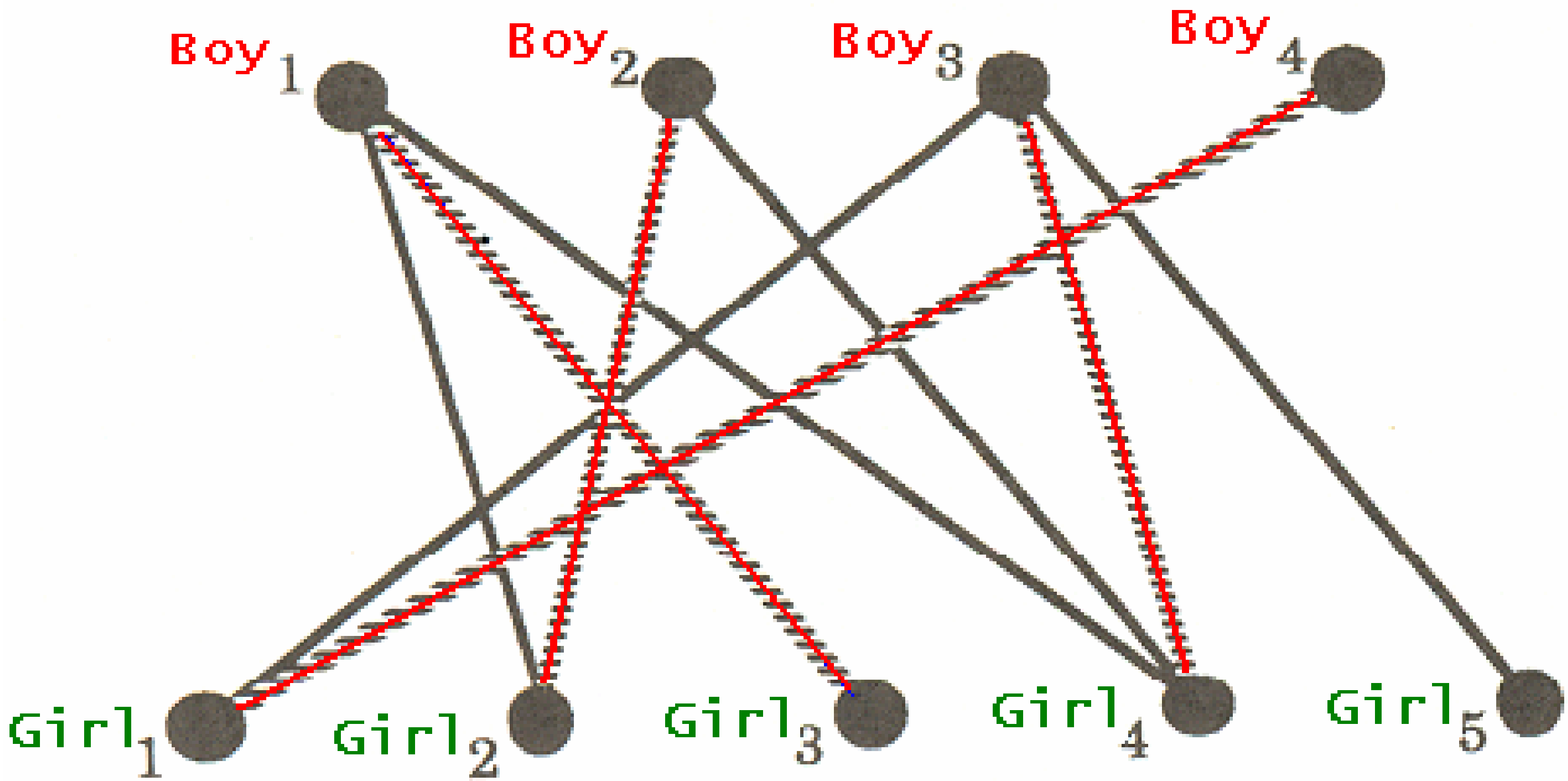
Assignment
Problem
分配問題



Matching & Assignment Problem (匹配)

The Marriage Problem (Hall - 1935)

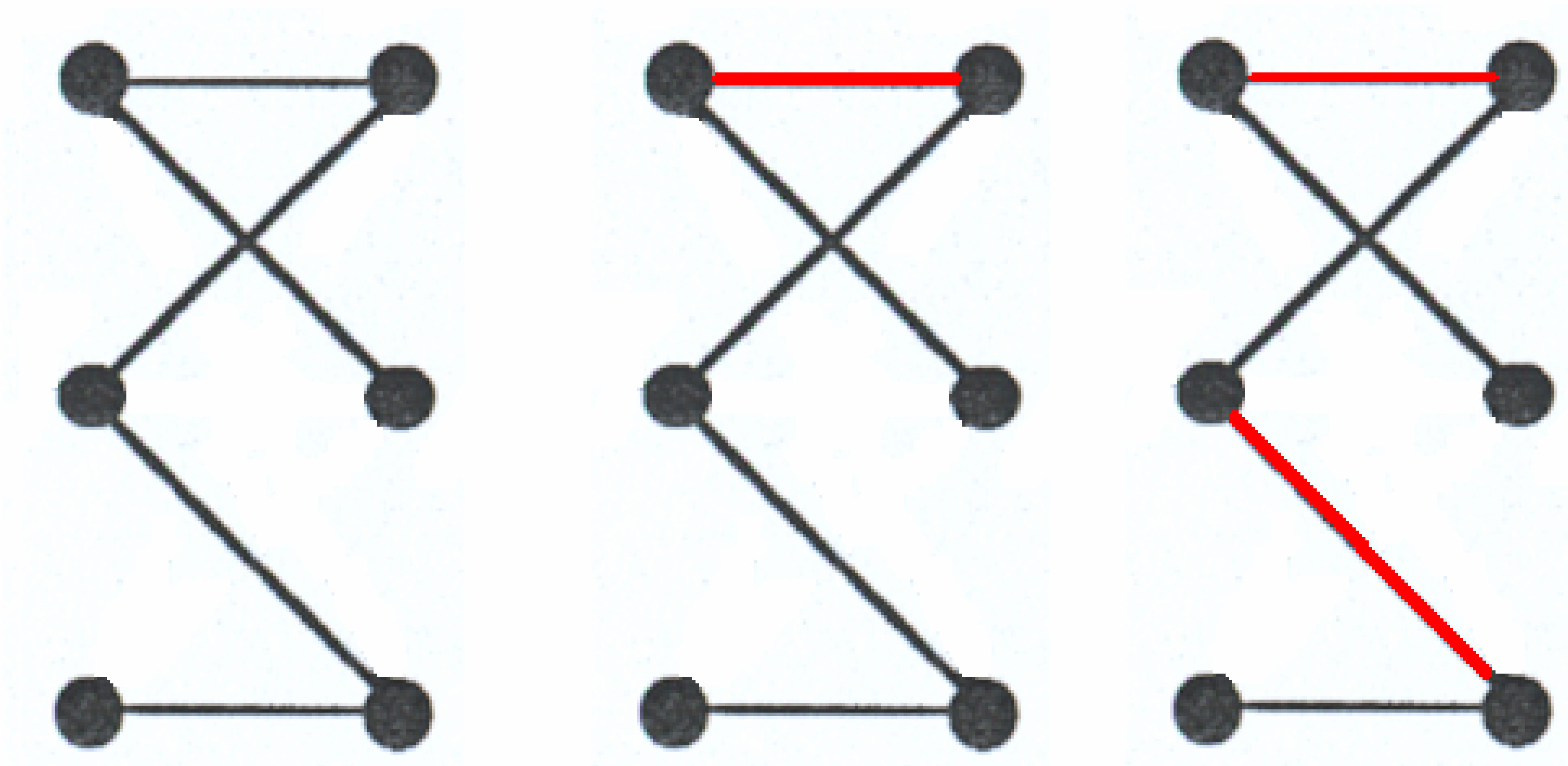
Each boy has several girlfriends.
How can each of them marry exactly ONE of his girlfriends?



One max matching solution - all 4 matches.

Matching & Assignment Problem（匹配）

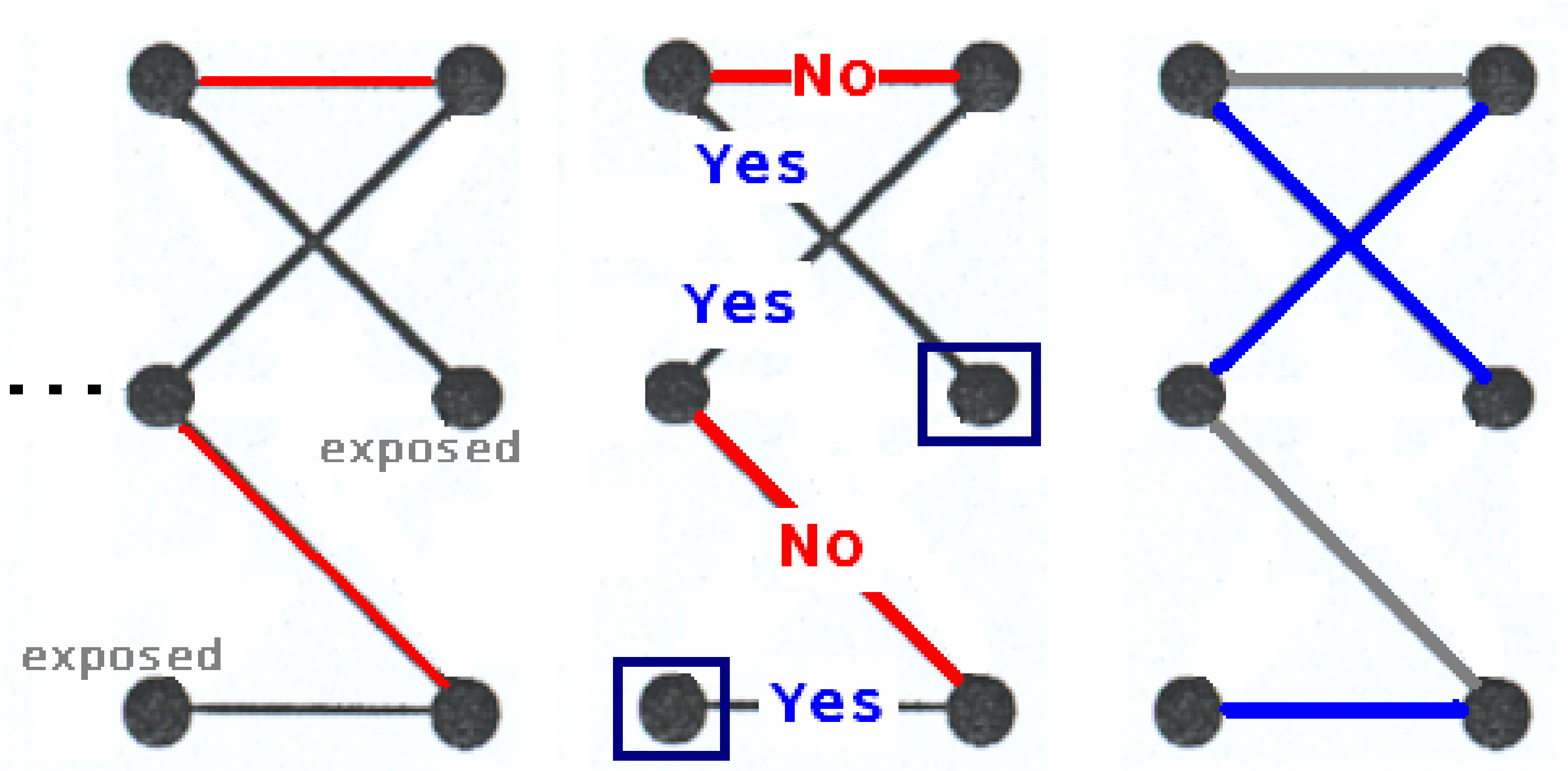
Theory. Hall's Marriage Theorem



Heuristic & SUB-OPTIMAL Solution |M|=2

Matching & Assignment Problem (匹配)

Algorithm. Alternating Chain Technique Berge-1957

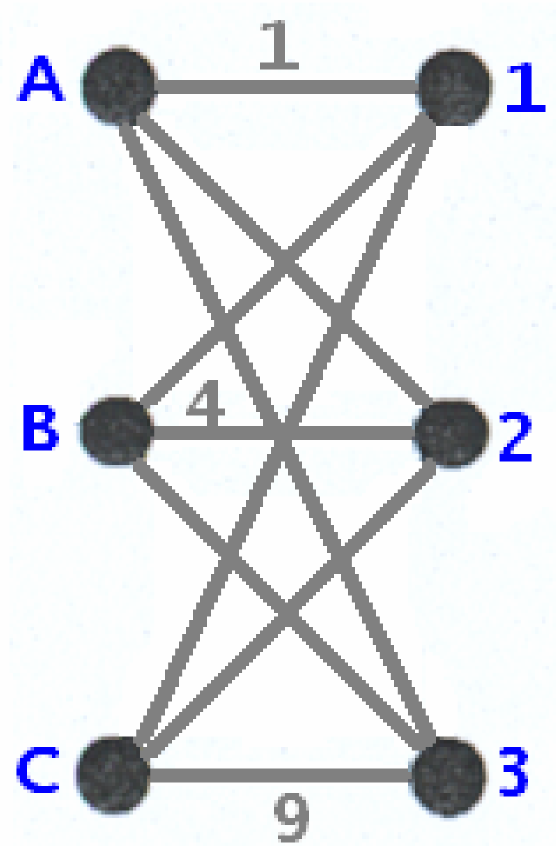


OPTIMAL (maximum matching) solution |M|=3

Matching & Assignment Problem (分配問題)

Application 1: Maximum value AP (最大值)

Application 2: Minimum cost AP (最小價)



	1	2	3
A	1	2	6
B	3	4	8
C	5	7	9

Value/Cost Matrix

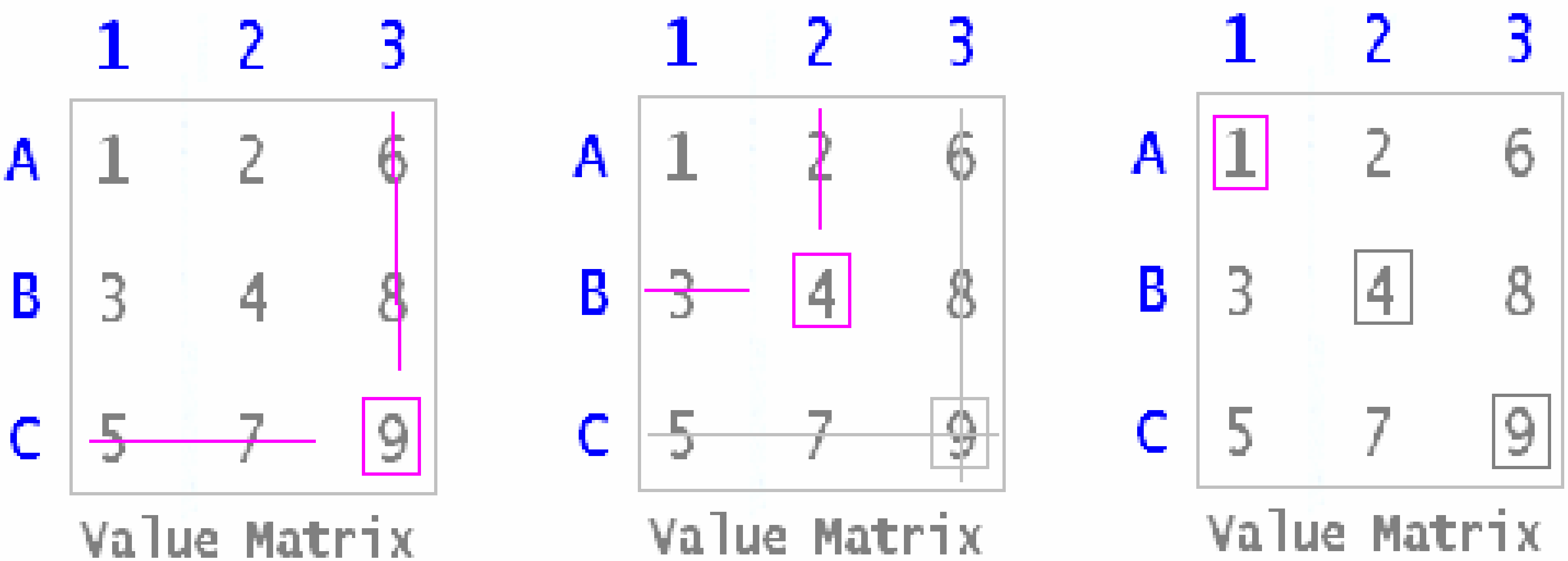
Example: { A-1, B-2, C-3 }
Total weight = 1+4+9 = 14

AP – 50 years anniversary

Matching & Assignment Problem (分配問題)

Application 1: Maximum value AP (最大值)

貪婪之啓發式方法 ***Greedy heuristics:***



Greedy heuristic gives {C-3, B-2, A-1} V=14

Optimal (max) solution:{A-1, B-3, C-2} V=16

Matching & Assignment Problem (分配問題)

Application 2: Minimum cost AP (最小價)

Algorithm:
Hungarian Algorithm -
substract constants from row or column

1	2	6	-1
3	4	8	-3
5	7	9	-5

0	1	5
0	1	5
0	2	4
	-1	-4

[0]	0	1
0	[0]	1
0	1	[0]

OPTIMAL (min) solution: {A-1, B-2, C-3} C=14

[NOTE: Same solution as "Max" heuristics!!]

Matching & Assignment Problem (分配問題)

Application 1: Maximum value AP (最大值)

1	2	6
3	4	8
5	7	9

10-C(i,j) =>

9	8	4	-4
7	6	2	-2
5	3	1	-1

5	3	0
5	4	0
4	2	0
-4	-2	

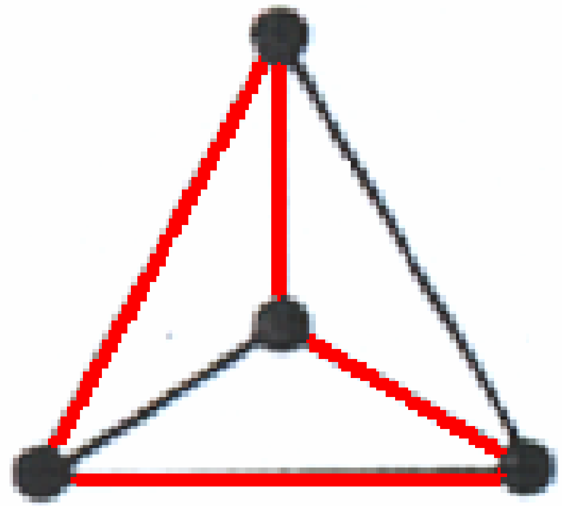
1	1	0
1	2	0
0	0	0

[0]	0	0
0	1	**[0]**
0	**[0]**	1

Optimal (max) solution:{A-1, B-3, C-2} V=16

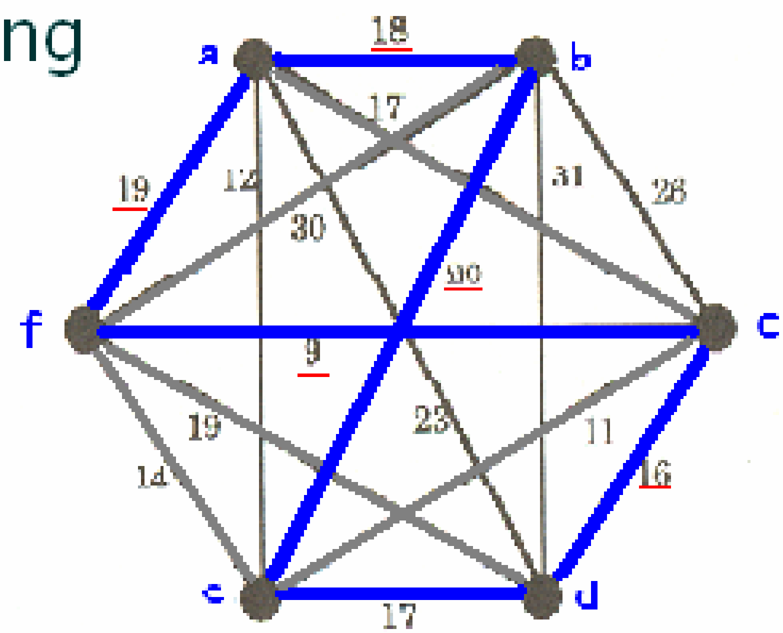
Graph
圖

Hamilton cycle
哈密頓圈

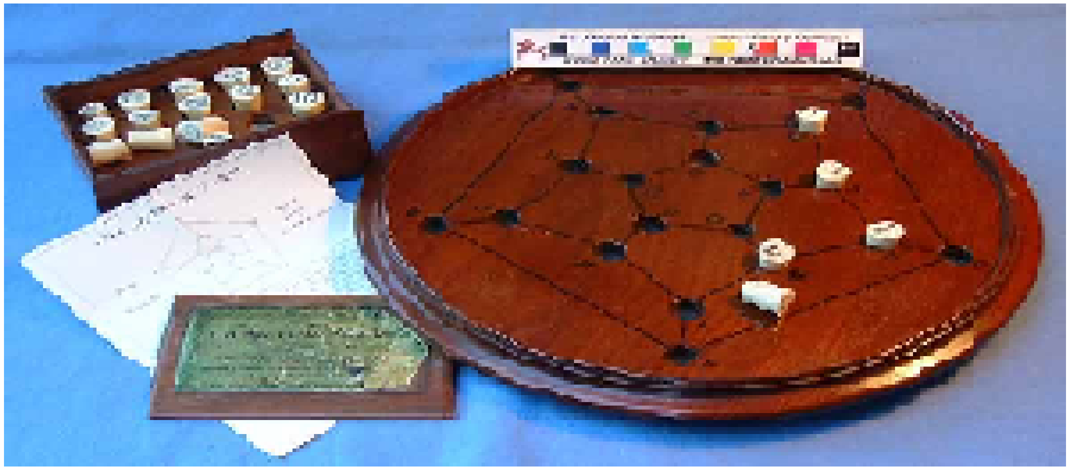


Network
網絡

Travelling Salesman Problem
(行進) 推銷員問題



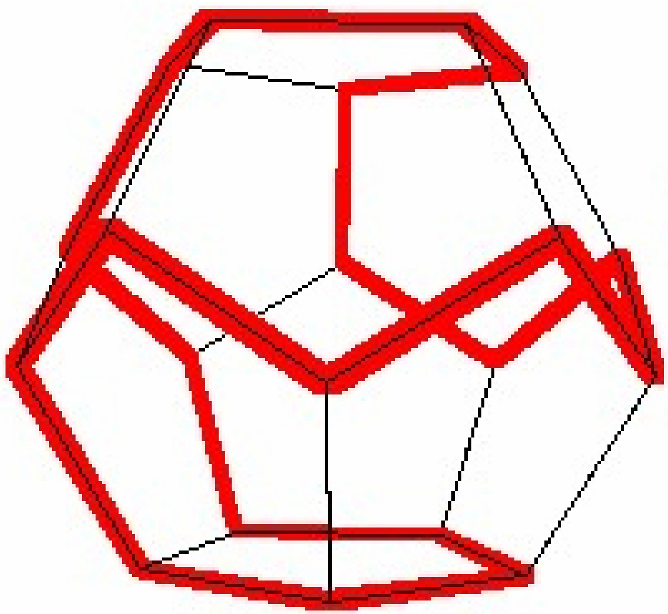
Hamilton Cycle & Travelling Salesman Problem

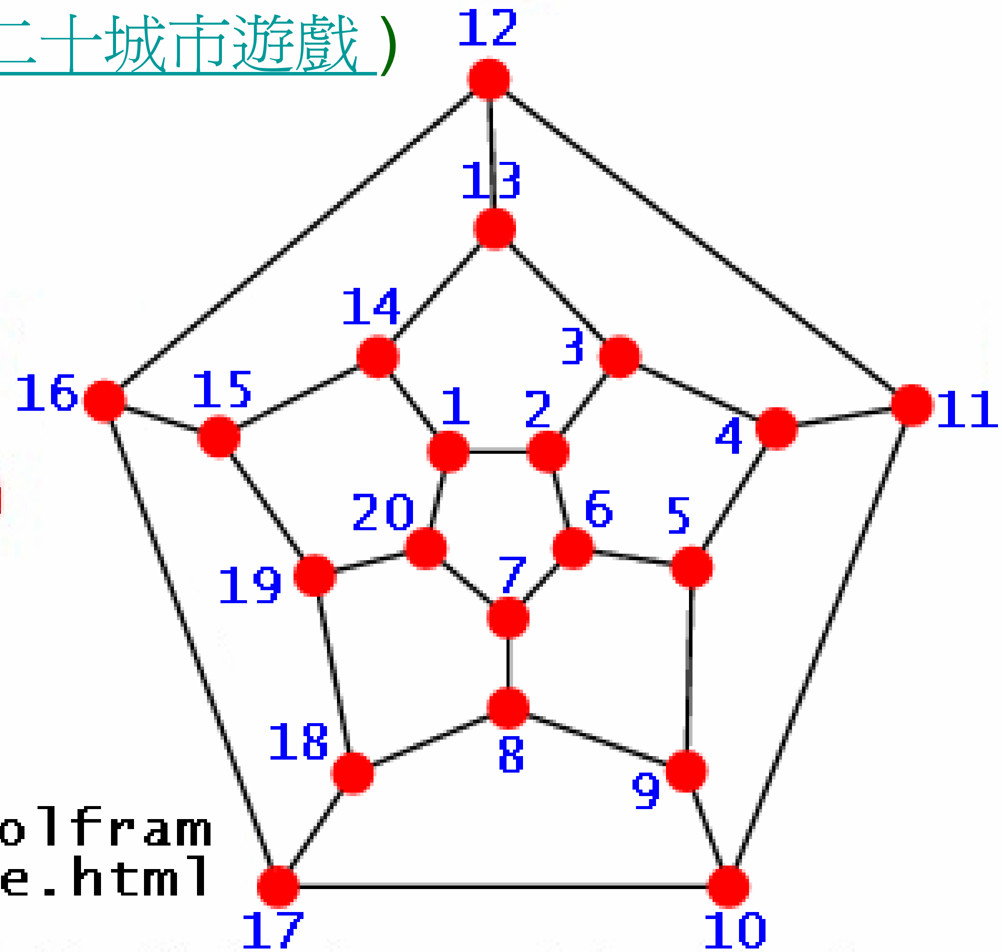


哈密頓圈(Hamilton Cycle) 是以所謂"周游世界問題"中提出來的. 1857 年, Sir. William Hamilton 設計了一個遊戲: 給了世界上的 20 個城市. 用一個代表地球的十二面体的 20 個頂點分別代表 20 個城市. 現在要求沿十二面體的遊戲(Icosian Game), 走過每個城市一次且僅僅一次, 最後回到出發點. 這個問題總結爲尋求圖中的一個圈, 它過每點一次且僅僅一次。

Icosian Game (二十城市遊戲)

十二面體



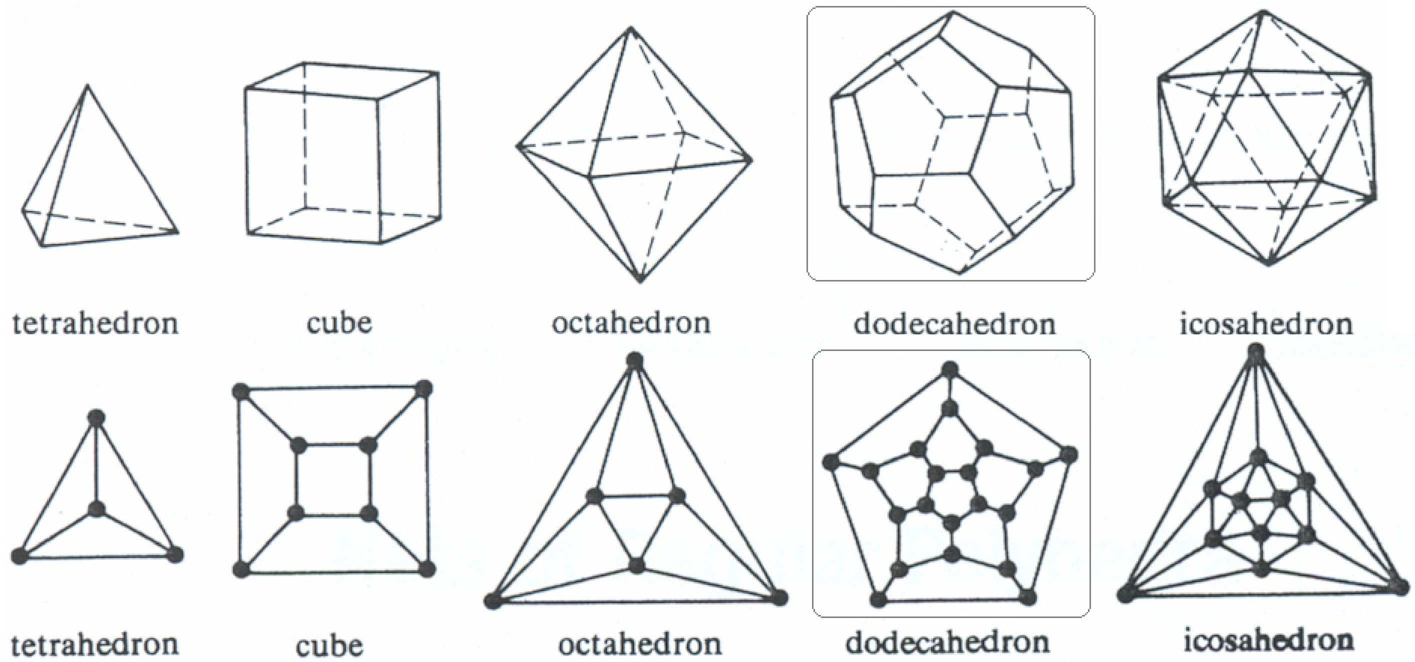


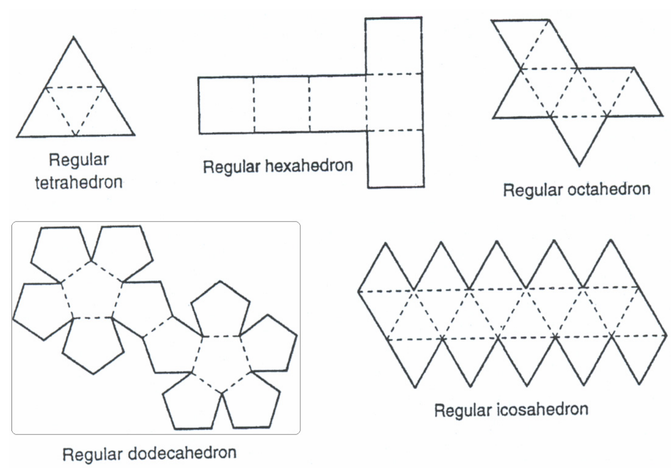
十二面體圖

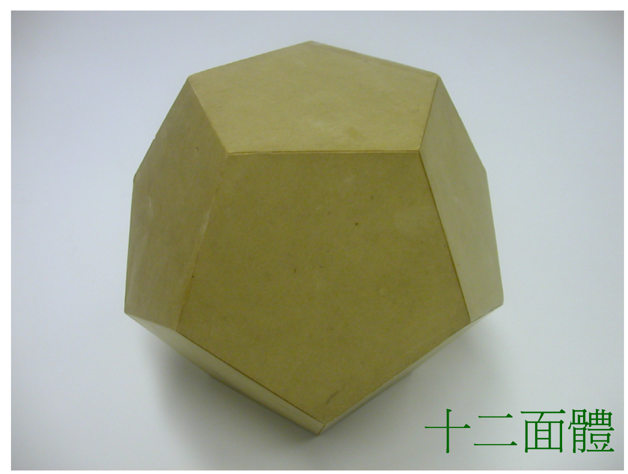
IcosianGame
http://mathworld.wolfram.com/IcosianGame.html

The Icosian game, also called the Hamiltonian game (Ball and Coxeter 1987, p. 262), is the problem of finding a Hamiltonian circuit along the edges of an dodecahedron, i.e., a path such that every vertex is visited a single time, no edge is visited twice, and the ending point is the same as the starting point (left figure). The puzzle was distributed commercially as a pegboard with holes at the nodes of the dodecahedral graph, illustrated above (right figure). The Icosian Game was invented in 1857 by William Rowan Hamilton. Hamilton sold it to a London game dealer in 1859 for 25 pounds, and the game was subsequently marketed in Europe in a number of forms (Gardner 1957).

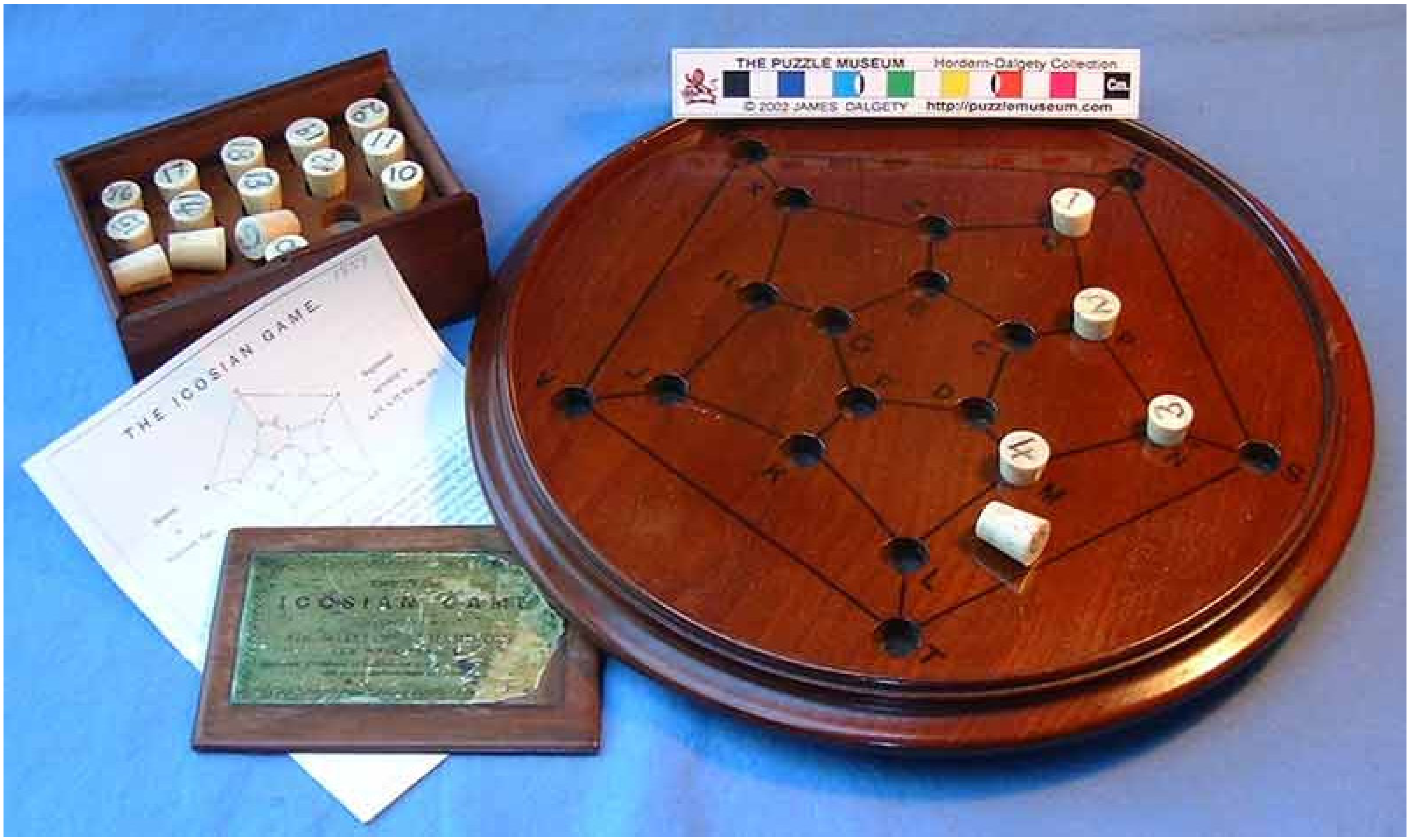
Platonic Solids, Graphs & Nets (cf. Plato’s Timaeus) 400BC



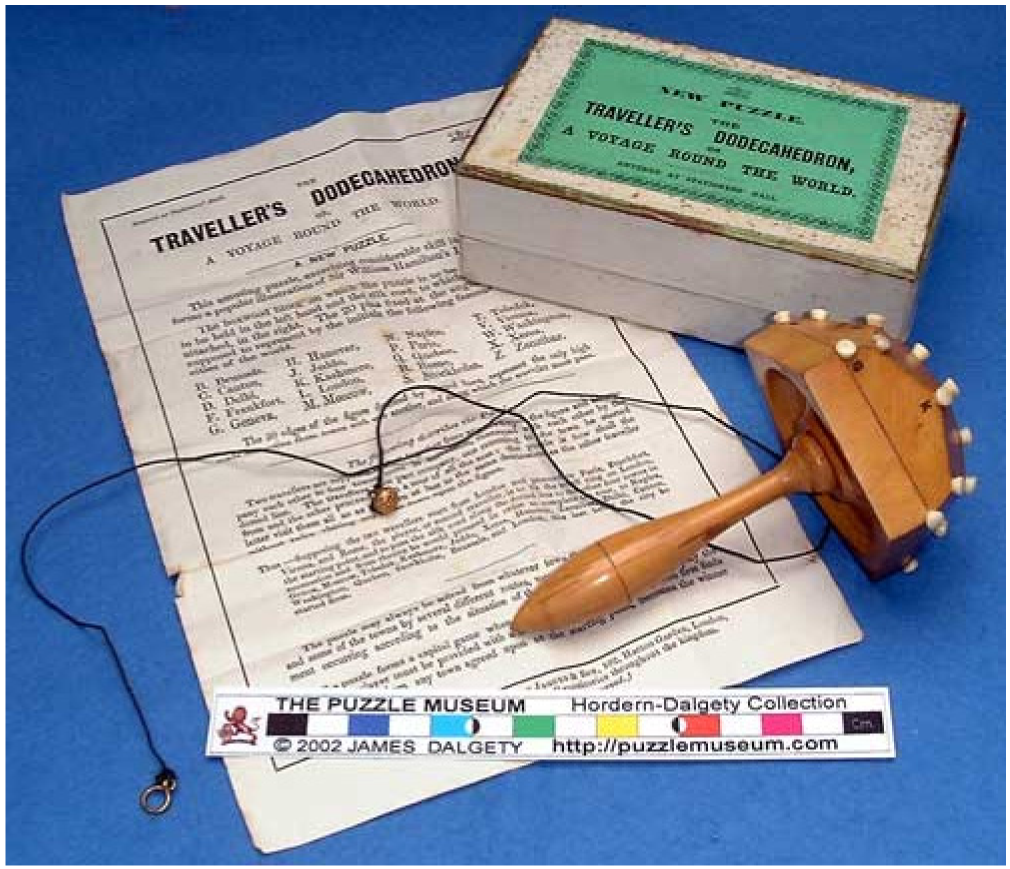




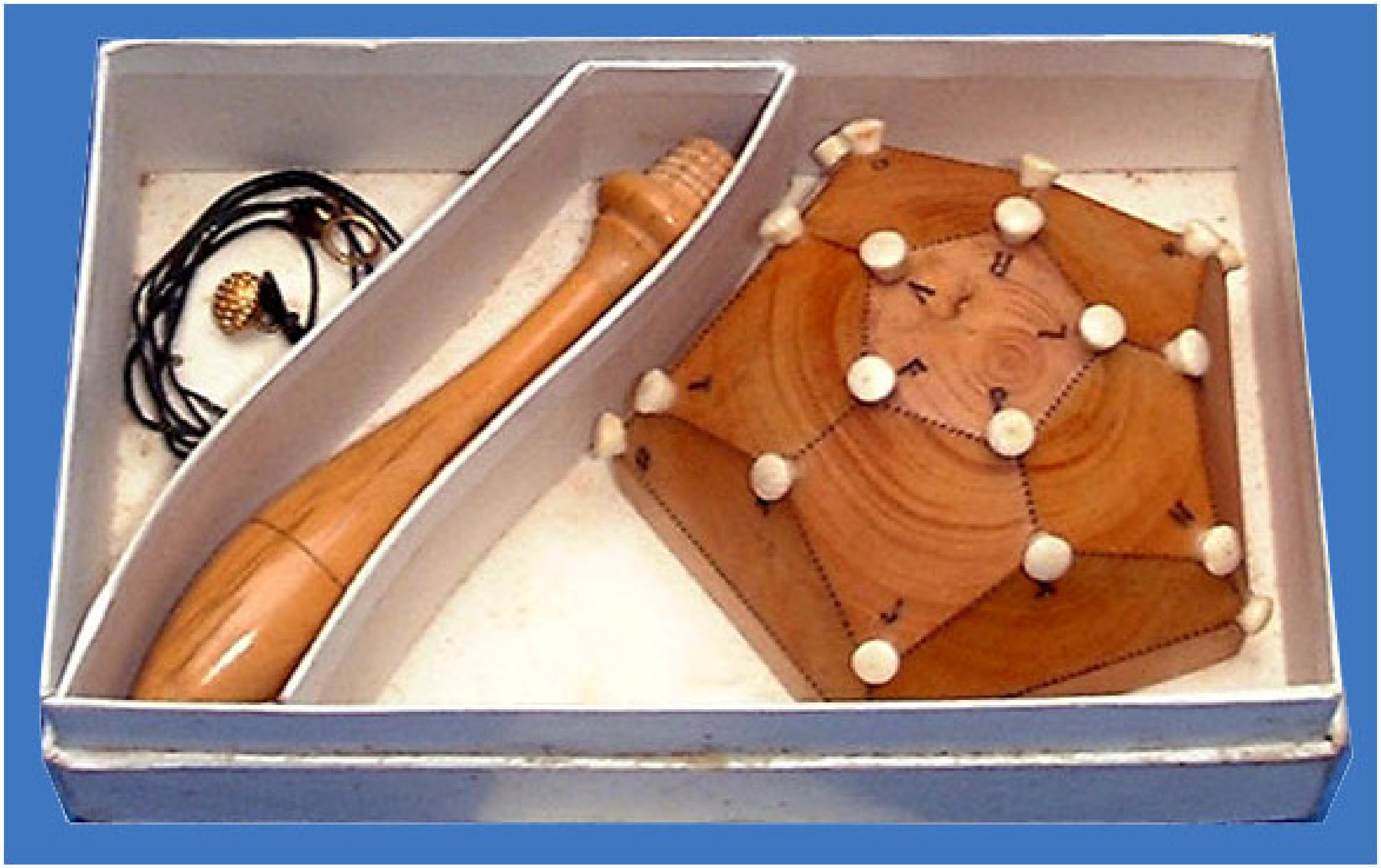
The original set of Icosian Game board in the Puzzle Museum



The Travellers Dodecahedron set (Icosian Game) in the Puzzle Museum



The Travellers Dodecahedron set (Icosian Game) in the Puzzle Museum

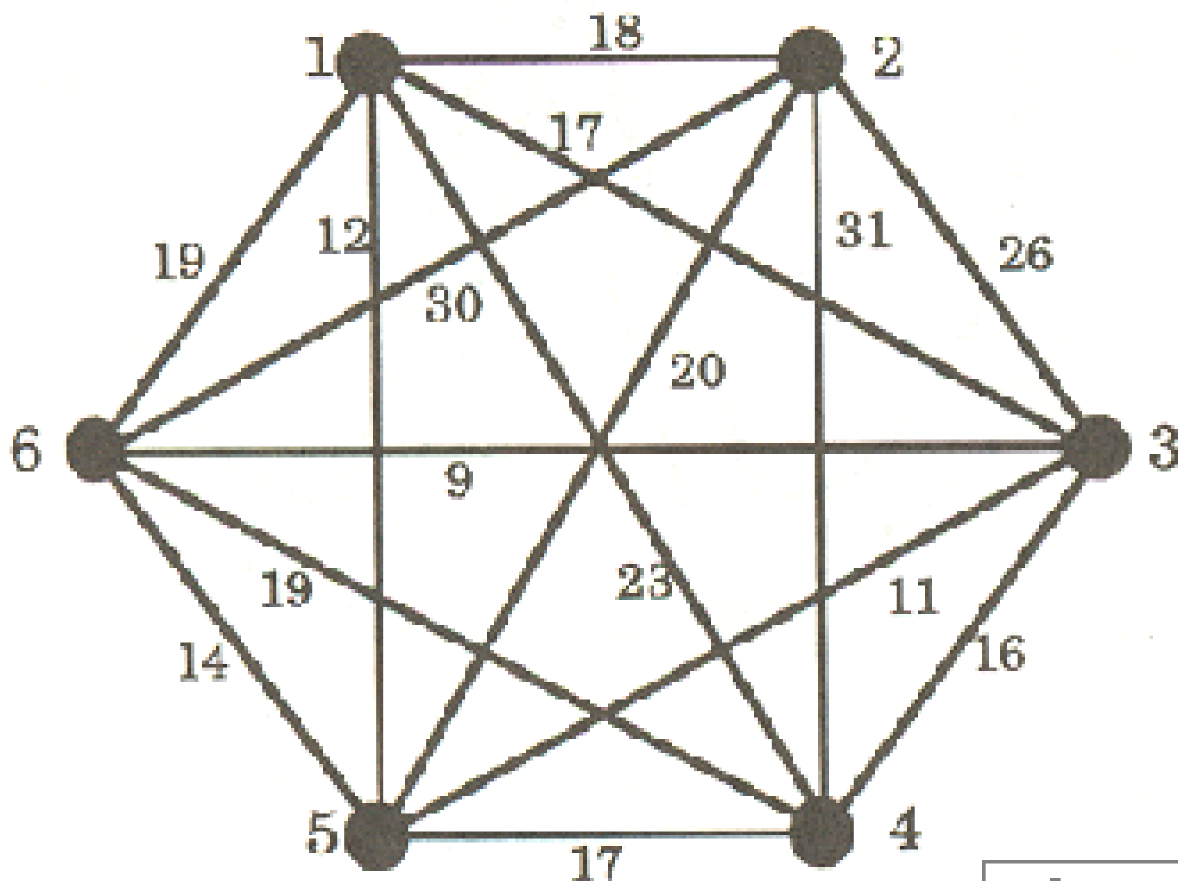


Hamilton Cycle & Travelling Salesman Problem

(行進)推銷員問題 - TSP

Application

TSP is to find a min-cost Hamilton cycle.

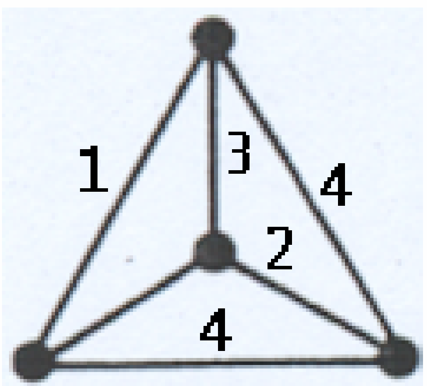


Min cost = 99

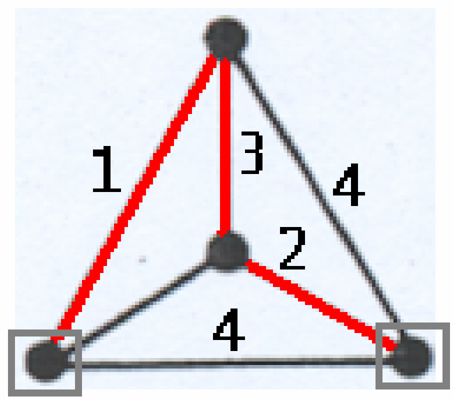
Hamilton Cycle & Travelling Salesman Problem

Approximate Algorithm (Christofides-1975)

Minimum spanning tree + weighted matching + Euler cycle. [within 50% over optimal cost]

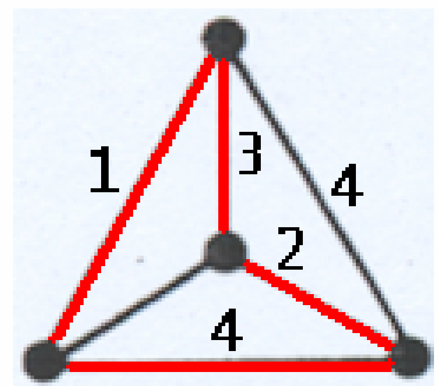


MST (cost=6)



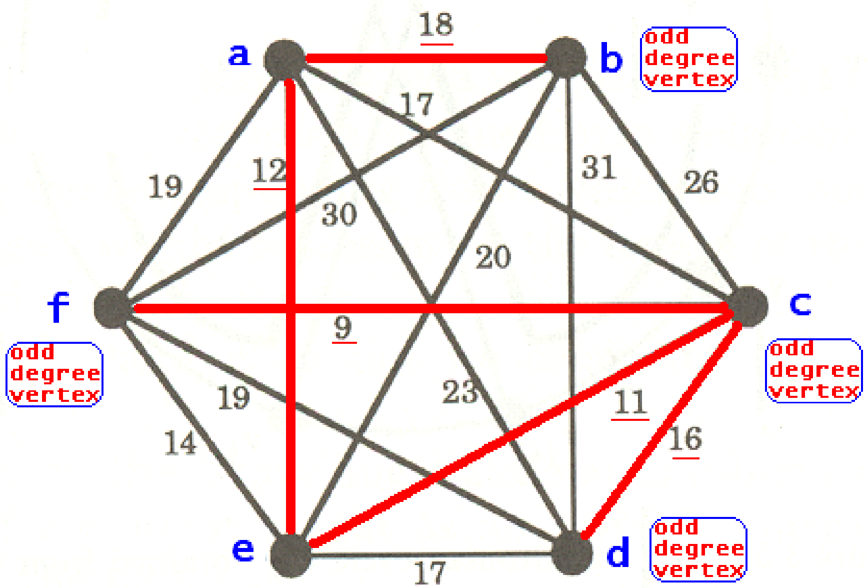
MWM (cost=4)

"approximate" solution

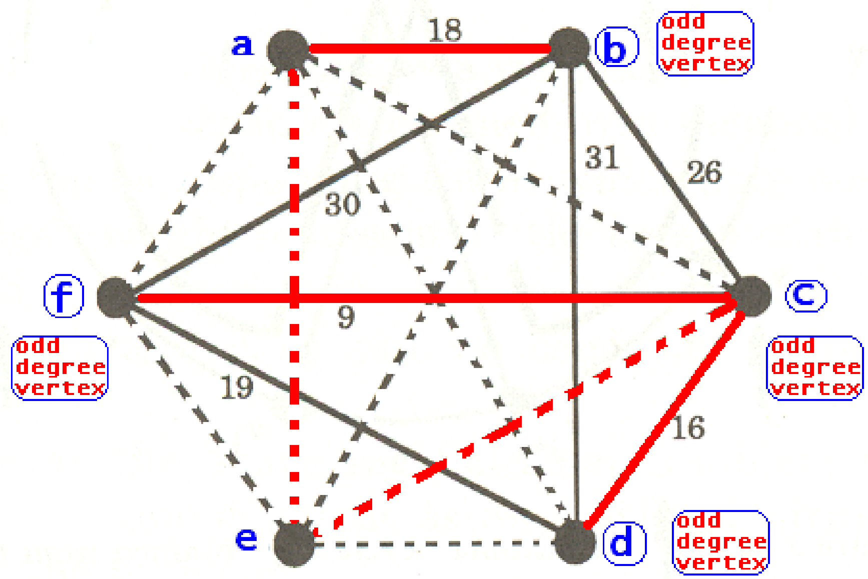


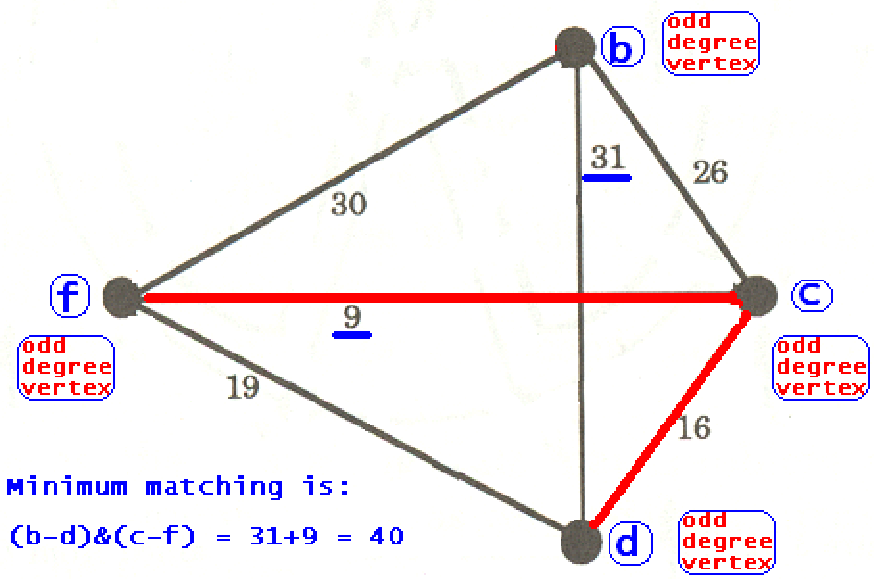
cost = 10

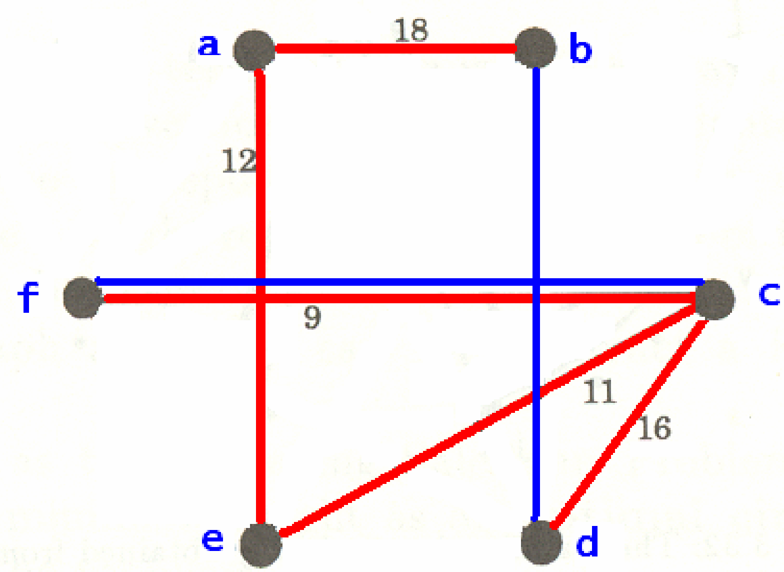
NOTE: In this case, "approximate" solution turns out to be optimal.



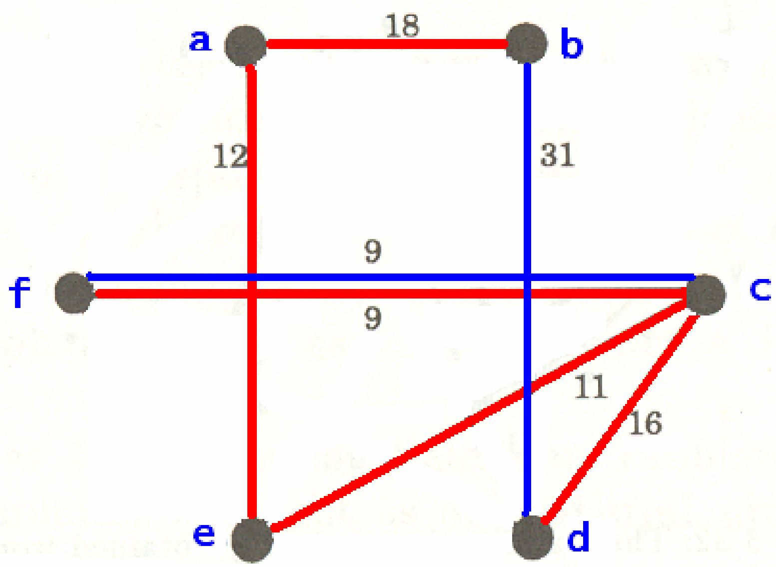
MST cost = 9+11+12+16+18 = 66





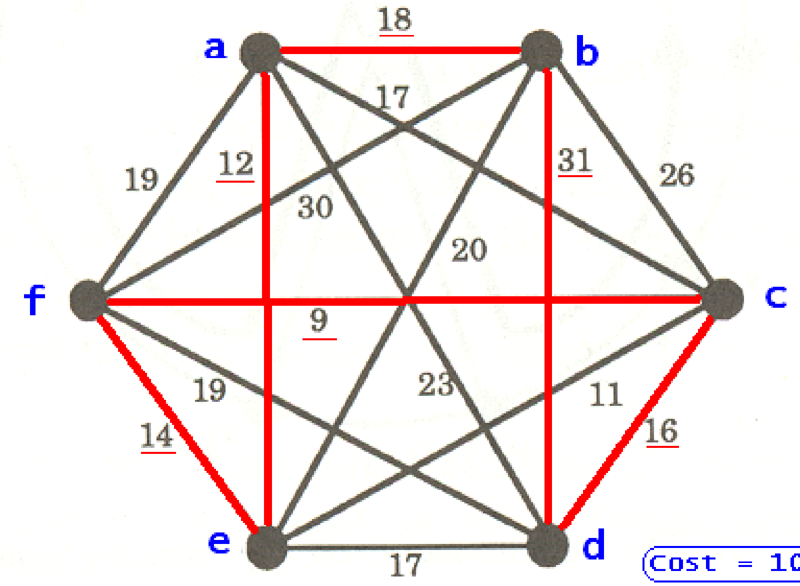


All vertices have even degrees.

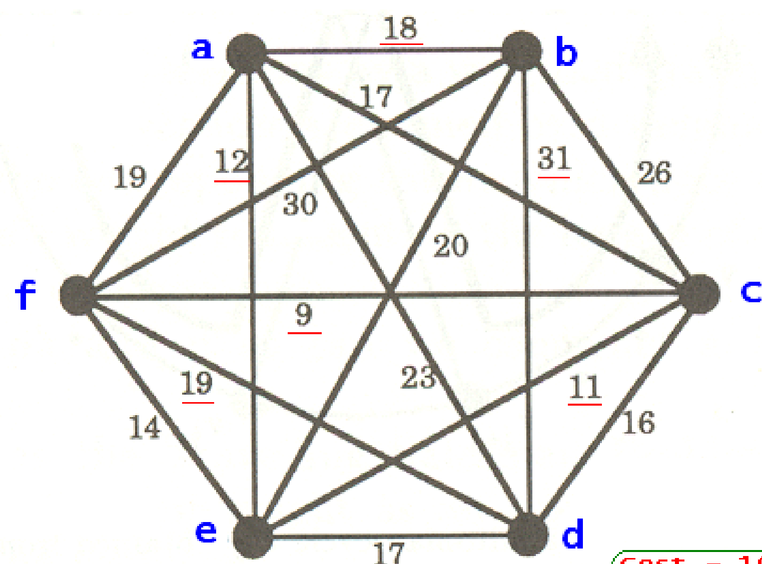


Euler cycle: a-b-d-c-f-c-e-a

Skipping the repeated vertice (c) gives TSP tour:
a – b – d – c – f – e – a



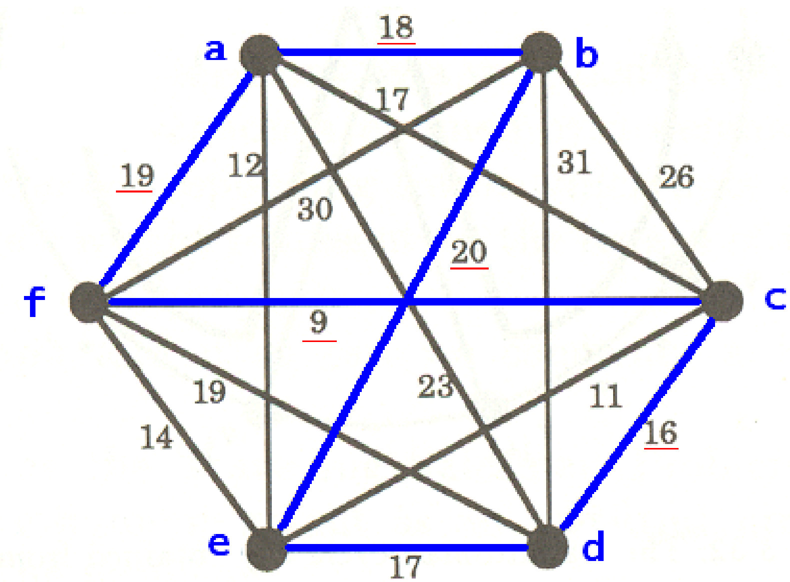
Cost = 100



Cost = 100

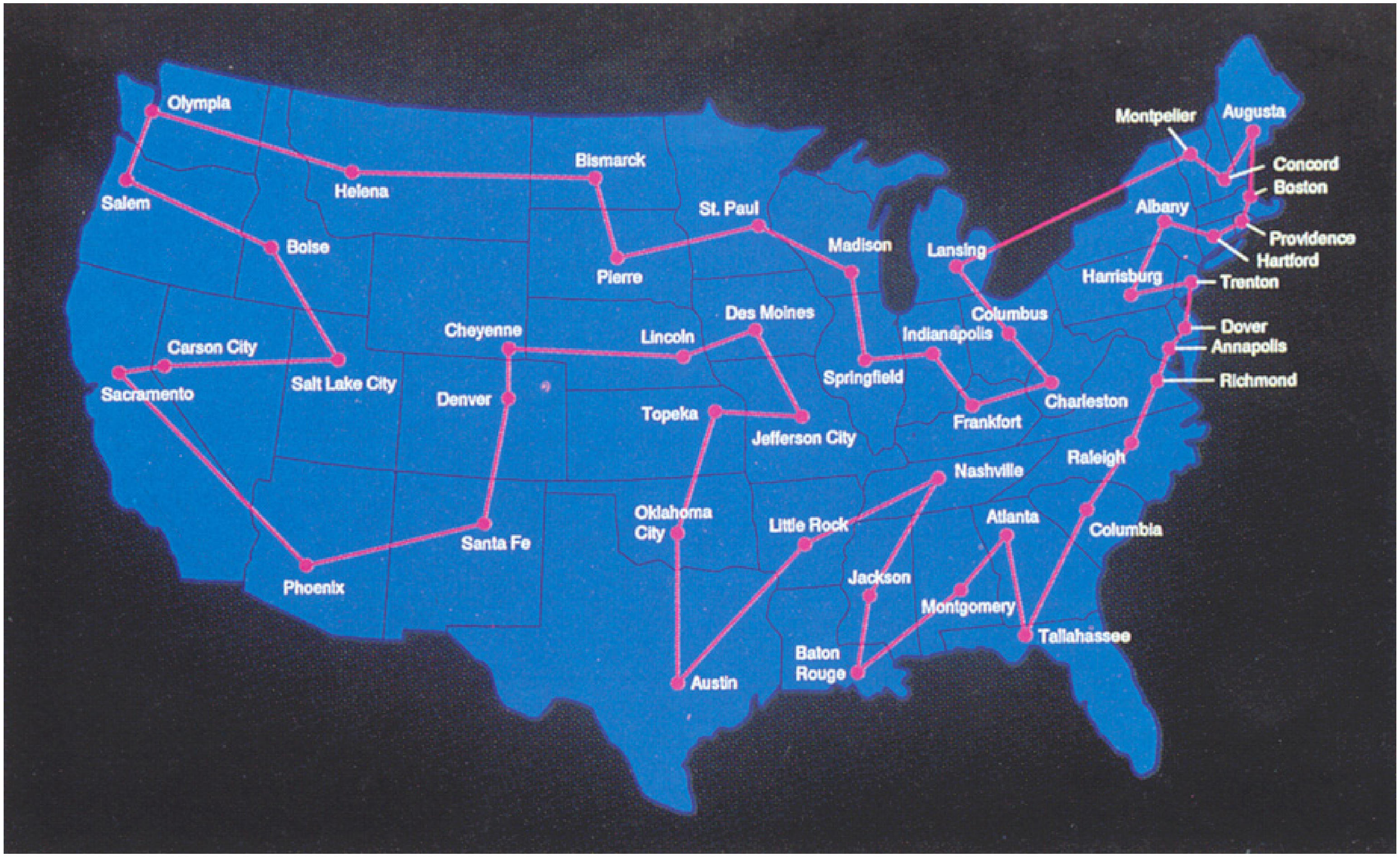
Nearest Neighbour (greedy) Heuristics:
9 + 11 + 12 + 18 + 31 + 19
(f) -- (c) -- (e) -- (a) -- (b) -- (d) -- (f)

Optimal TSP tour (cost = 99):
a - b - e - d - c - f - a



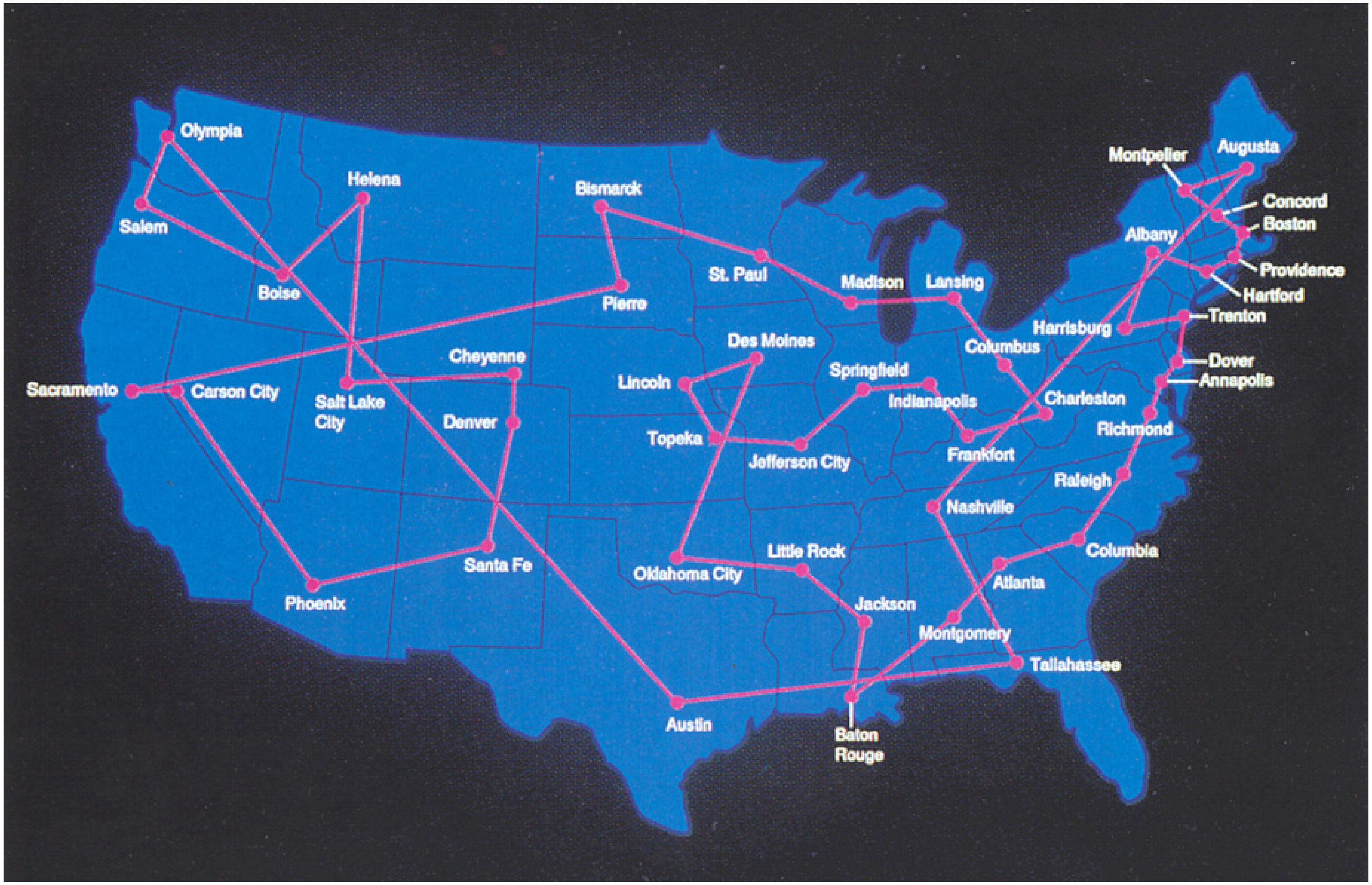
An Illustration of Travelling Salesman Problem: Optimal Tour Solution [12000 miles for 48 cities]

(source: Tannenbaum & Arnold)



An Illustration of Travelling Salesman Problem: Heuristic Tour Solution [14500 miles for 48 cities]

(source: Tannenbaum & Arnold)



THANK YOU

CREDITS & ACKNOWLEDGEMENT

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田豐、馬仲蕃編著．科學出版社(1987).

AND - Various Internet sources.

Special Thanks – Colleagues of Mathematics Department at
The University of Hong Kong

香港大學數學系主辦公開講座

數趣漫話

之

運籌學漫遊

圖論與網絡的數學導賞

講者：朱進強博士

日期 2006年11月11日（星期六）

時間 10:00－11:30 am

地點 香港大學黃麗松演講廳

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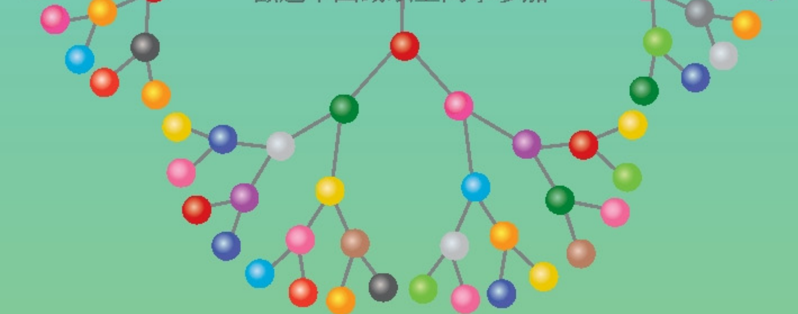

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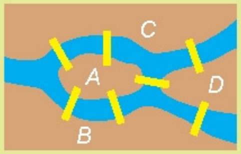

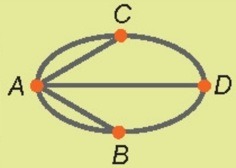

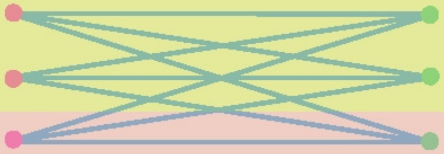

這個講座，將引領各位聽眾漫遊運籌學的勝景——網絡模型。

網絡模型，建基於圖論及最優化概念之上，以數學家的大膽直覺及細密思維巧築而成，其中的「理論—啟發—算法—啟發—應用—啟發—理論」環迴路徑，更是最優化建模、以至應用數學的優美典範。她在多方面的應用實例，亦各具姿采，讓人目不暇給。

你，可有興趣共赴一遊？

數趣

朱進強博士

康乃爾大學學士及碩士、哥倫比亞大學博士。現任教於香港大學數學系，曾著述論文數十篇。主要研究方向為運籌學，以對本港實際問題的應用知名。歷任各工業、商業、服務行業機構之顧問。