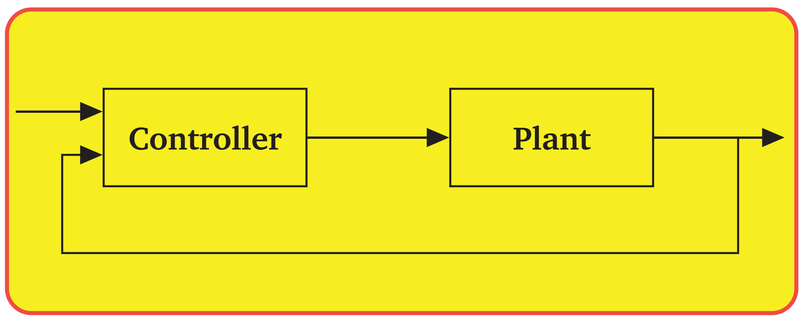
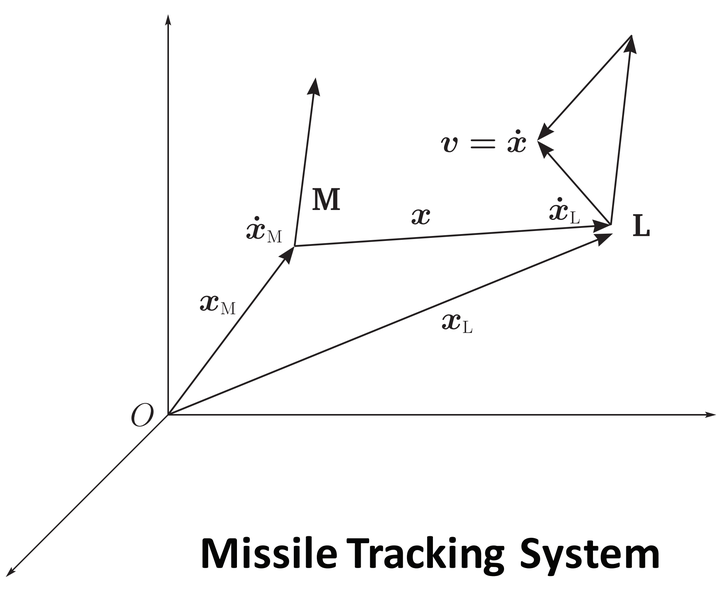
Near-Optimal Controls for Delay

Dr. S.P. Yung, Department of Mathematics

Optimal control systems appear in a lot of engineering and financial problems. Its aim is to make the system under investigated performs to its optimal by picking the right control function. These "right" control functions are labelled as "optimal control functions", they could correspond to the level of various inputs for an engineering problem or the amount of investments in each individual stock in a financial portfolio, or even the dosage of nutrients to make the populations of various species in an environmental problem optimal.



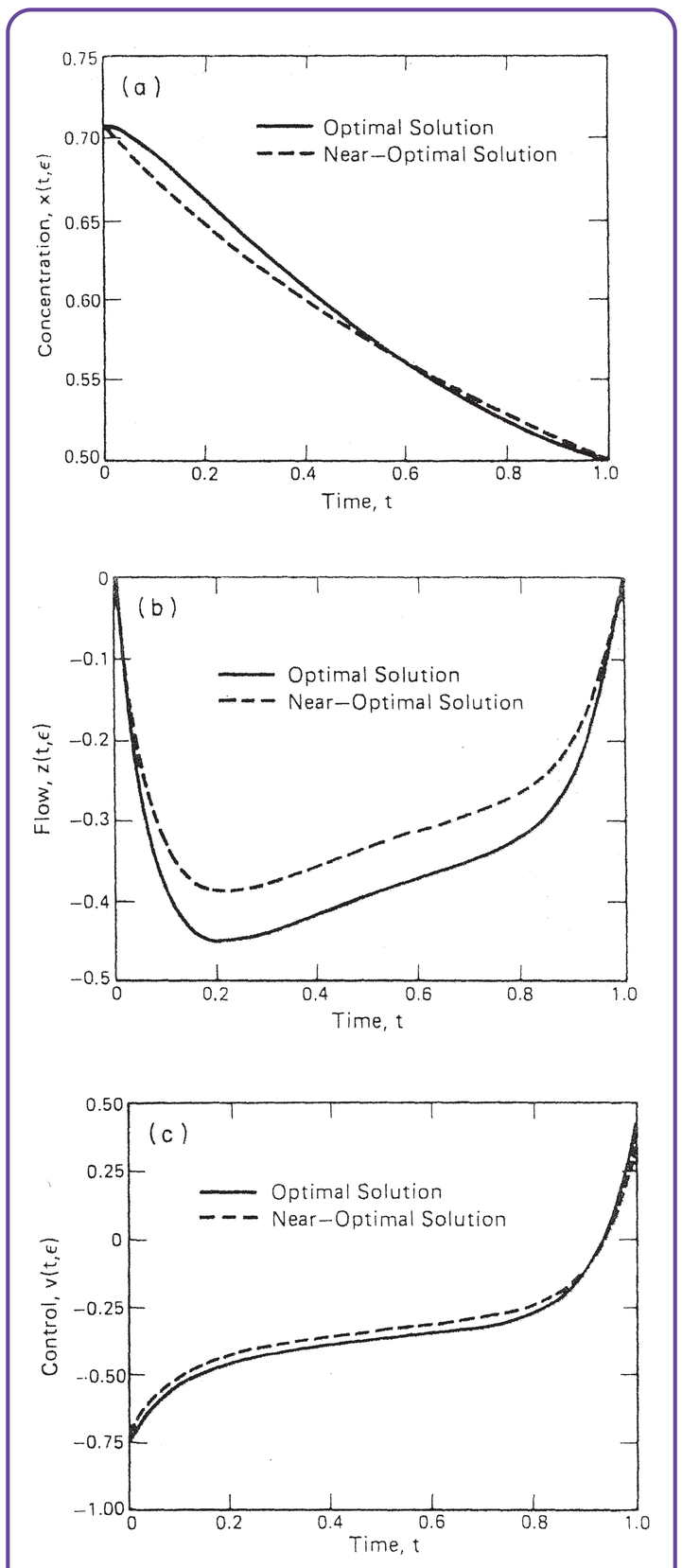
Dr. S.P. Yung has been studying optimal control problems that have time-delay effects with his co-workers. These time-delays come from the time-lags responses among the various components in the system. When present, they are well-known to be able to change the behavior of a system dramatically, from stable to unstable, from optimal to non-optimal.



Missile Tracking System

In this sense, they cannot be ignored, and we aim at identifying optimal control functions for these time-delay problems. Since small errors frequently occur in the calibrations of parameters for a system, the optimal control function may not be robust with respect to these small parameter changes. Near-optimal control functions are thus more desirable in practice. It is our goal to search for methods in identifying near-optimal control functions for time-delay control systems. The problems that we are solving are nonlinear and nonconvex, which impose another difficulty in the investigation.

The way that we search for near-optimal control functions is through setting up necessary conditions and sufficient conditions that they are required to satisfy. We use a pre-determined constant ϵ to gauge how far away from the true optimal the near-optimal control is allowed to be differed. Under this ϵ, a necessary condition is constructed in the form of maximum principle. This maximum principle is also a sufficient condition for the near-optimality when the system is convex. When the system is not convex, we have successfully come up with a sufficient condition for the near-optimality in the form of a Riccati inequality. So when both the necessary condition and the sufficient condition are satisfied, the control function must be near-optimal for the system.



Near-Optimal Solution for Various System

The framework that we are putting down is very general and suits a lot of problems that appear in practice. Our established conditions help readily in finding the near-optimal control functions for a lot of problems. The Riccati inequality sufficient condition is very convenient to use since its numerical schemes are abundant whenever needed.

Further Reading:

Richard C.H. Lee, C.W. Wong and S.P. Yung, ϵ-Suboptimal Control for Time-Delay Systems, in preparation, 2016.