Guide to Writing Mathematics

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1 Preface

This guide is developed under the Teaching Development Grant (TDG) project entitled 'Promoting Teaching and Learning of Professional Writing in Mathematics' (Principal Investigator: K. H. Law, Department of Mathematics; Co-investigators: S. Boynton and C. Tait, Centre for Applied English Studies) at The University of Hong Kong. The first version was released in January 2015 upon completion of the project, while continual updates and enrichments to the contents are expected.

All mathematics students have to *write mathematics* to some extent, but then unlike in the teaching of *other* languages (mathematics is certainly a language in its own right!) there is seldom a course that specifically teaches students how to write properly in the mathematical language. Furthermore, the way university mathematics teachers expect students to write is often different from how students used to write in mathematics homework or examinations in secondary school. One of the main purposes of the project is therefore to bridge the gap and raise students' awareness of the importance of proper mathematical writing.

It is hoped that this guide will serve as a reference to students on how to write mathematics, as well as a resource for both mathematics and English teachers.

2 Basic Principles

Although there is no well-defined rules in writing mathematics (unlike the subject of mathematics itself in which everything seems to be rigorously defined), there are some basic elements which should be remembered.

2.1 Using Complete Sentences

One of the misconceptions students have about writing mathematics (which probably arises from their writing habits in secondary school) is that the writing should be composed mostly of equations and mathematical symbols. This is not true at all. By contrast, a piece of mathematical writing should contain mostly *words*, supplemented by equations and mathematical symbols. Take any mathematics textbook to verify this. Even in their worked examples, the solutions contain mostly words.

One of the very first basic principles in writing mathematics is therefore to write *structurally* in complete sentences. Take what you are reading now as an example. You can see that the whole subsection on 'using complete sentences' is divided into several *paragraphs*, each divided into several sentences. Each sentence is a *complete sentence.*

There can be equations and mathematical symbols, but still they form a complete sentence when properly read. Some examples are as follows.

Sentence containing equations and mathematical symbols	How it is read as a complete sentence
Hence $x + 1 > 3$.	Hence x plus 1 is greater than 3.
Since $x > 2$, it follows that $x^2+3x > 2^2+3(2) = 10$.	Since x is greater than 2, it follows that x squared plus three x is greater than two squared plus three times two which is equal to 10.
Thus $n \neq 0\ \forall n \in A$.	Thus n is not equal to 0 for all n that belongs to A.

On the other hand, the following lists some examples which are *not* complete sentences and hence should not appear in a piece of mathematical writing.

- Since x is positive.
- If this is not true.
- When two triangles are similar.

2.2 Starting Sentences with Capital Letters

This sounds pretty much natural, as this is how you write an English passage. However, when it comes to writing mathematics, sometimes we may be tempted to start a sentence with a small letter because it is a symbol or a variable with which we want to use to start the sentence. Here are a couple of examples, and how they may be modified:

Sentence starting with a small letter	How it may be modified
p is not a prime number if it is divisible by 3 and greater than 3.	If the number p is divisible by 3 and greater than 3, then it is not a prime number.
$x^2 + x + 1 = 0$ has no real root since its discriminant is negative.	The equation $x^2 + x + 1 = 0$ has no real root since its discriminant is negative

Some even suggest that we should never start a sentence with a symbol or a variable, even if that is in capital letter. In that case similar modifications as above may be applied.

2.3 Commas between Variables

We often separate variables by commas, for instance

- Let a, b, c be positive real numbers. Then $\dfrac{a+b+c}{3} \geq \sqrt[3]{abc}$.

However, consider the following sentence:

- In addition to p, q is also a prime number.

What's the problem? As we read, one naturally sees 'in addition to p, q', so we expect that p and q have some property, and apart from them there is yet another number with the same property. But then this is not what the sentence intends to convey. This is somehow related to the previous subsection when we mentioned that we should not start a sentence with a variable in lower case — it turns out that starting a *clause* with a lower case can be confusing too when the previous clause also ends with a lower case variable.

Again, we may apply some modification to make the sentence more readable:

- In addition to p, the number q is also prime.

Here are two more examples. Can you rewrite them?

- Since $x > 1$, $x - 1 > 0$ and so $\sqrt{x-1}$ is well-defined.
- As a quadratic equation in x, $x^2y + xy^2 + xy + 1 = 0$ has discriminant $(y^2 + y)^2 - 4y$.

3 The Use of English

While the use of English does not affect the actual mathematical reasoning, it is still an important aspect students should be aware of. A piece of mathematics written in good English helps the reader follow the argument more easily. The examples here are taken out of their original contexts for simplicity. A lot of these mistakes are made probably because most of the students are not native English speakers. We try to split the errors into several categories, but often one will find that some of the examples fit under multiple categories.

3.1 Grammatical Errors

Grammar refers to the way words are put together to form phrases and sentences. A grammatical error is when these rules for grammatical structure are broken.

3.1.1 The use of articles

The definite article *the* indicates that its noun is a specific one which the reader should know. There is only one binomial theorem and therefore we say 'the binomial theorem'. On the other hand when we define x to be some non-negative number, very often it does not tell the reader which specific number we want it to be, and thus the indefinite article 'a/an' should be used instead. Whether 'a' or 'an' should be used depends on the pronunciation of the word or letter following it.

Wrong	Correct	Comments
Construct ***a*** $m \times n$ table.	Construct ***an*** $m \times n$ table.	When the letters m and n are pronounced there is actually a vowel sound /e/ at the beginning of each letter. This means that the article 'an' should be used.
1. By ***binomial theorem***, we have... 2. By ***the Pythagoras' Theorem***, ABC is a right-angled triangle.	1. By ***the binomial theorem***, we have... 2. By ***Pythagoras' Theorem***, ABC is a right-angled triangle.	When referring to a specific theorem (e.g. the Fundamental Theorem of Calculus) use 'the' (the definite article). However, when referring to a theorem from a named person use the zero article.
By (a), ***the*** $\boldsymbol{g}$ is continuous.	1. By (a), $\boldsymbol{g}$ is continuous. 2. By (a), ***the function*** $\boldsymbol{g}$ is continuous.	When naming something with a letter (e.g. g) use the zero article. When using the noun function use the definite article (e.g. the function g).
1. Let x be ***non-negative number***. 2. Let x be ***the non-negative number***.	1. Let x be ***a non-negative number***. 2. Let x be ***non-negative***.	When using the phrase 'negative' or 'non-negative number' use the indefinite article because 'a' means any number. When using 'non-negative' as an adjective then use the zero article.
Let x be ***a positive***.	Let x be ***positive***.	Use the adjective form and not the noun form.

3.1.2 The difference between singular and plural

In general, most nouns have more than one form depending on the corresponding quantity. If there are more than one object, the plural form is required; whereas the singular form is used if there is only one.

Wrong	Correct	Comments
Throwing two fair ***dices*** once, we have $6 \times 6 = 36$ different possible outcomes.	Throwing two fair ***dice*** once, we have $6 \times 6 = 36$ different possible outcomes.	The singular form is 'die', the plural form is 'dice' (without 's'). In modern English sometimes 'dice' is also accepted as the singular form.

Wrong	Correct	Comments
Every square are rectangles.	1. Every square ***is a rectangle***. 2. ***All squares*** are rectangles.	1. Every + singular form of noun. 2. All + plural form of noun.
Assume $P(k)$ is true for some positive ***integers*** k, i.e. $k(k+1)$ is divisible by 2.	Assume $P(k)$ is true for some positive ***integer*** k, i.e. $k(k+1)$ is divisible by 2.	There is only one integer here (namely k); therefore it should be in singular form.
Let A be a square ***matrice***.	Let A be a square ***matrix***.	'Matrices' is the plural form of 'matrix'.
Let A be the ***vertice*** of the pyramid.	Let A be the ***vertex*** of the pyramid.	'Vertices' is the plural form of 'vertex'.
$(0,0)$ is the only local ***maxima***.	$(0,0)$ is the only local ***maximum***.	'Maxima' is the plural form of 'maximum'.

3.1.3 The different verb forms

Similar to nouns, most verbs have more than one forms, and you need to be aware of which ones are correct and which ones are wrong.

Wrong	Correct	Comments
1. There ***exists*** real numbers m and n such that $m > n$. 2. There ***exist*** a real number m such that $m > 1$.	1. There ***exist*** real numbers m and n such that $m > n$. 2. There ***exists*** a real number m such that $m > 1$.	Make sure the subject agrees with the verb. Whether to use 'exists' or 'exist' depends on whether the noun that follows is in singular or plural form.
There ***exist*** no integer k such that $3k = 2$.	There ***exists*** no integer k such that $3k = 2$.	'Integer' in this example is singular.
Replace a by $-a$, we have $f(-a) = (-a)^2 = a^2 = f(a)$.	1. ***Replacing*** a by $-a$, we have $f(-a) = (-a)^2 = a^2 = f(a)$. 2. ***If we replace*** a by $-a$, we have $f(-a) = (-a)^2 = a^2 = f(a)$.	'Replacing' has the same meaning as 'If we replace'; '-ing' clauses can go at the beginning of a sentence and are a dependent clause.

Wrong	Correct	Comments
Hence we get $x < 0$, ***contradicts to*** (1).	1. Hence we get $x < 0$, ***which contradicts*** (1). 2. Hence we get $x < 0$, ***which is a contradiction to*** (1).	The verb form 'contradict' does not have a dependent preposition, i.e. 'contradict something'. The noun form 'contradiction' does have a dependent preposition 'to'. The relevant phrase is 'which is a contradiction to'.
Thus ***contradiction***.	1. Thus ***a contradiction occurs***. 2. Thus ***there is a contradiction***.	In the wrong version there is no verb form in the sentence. The two correct sentences contain a verb.

3.1.4 Verb-to-be and verb-to-do

Two of the most frequently used verbs in English are the verb-to-be and the verb-to-do. They are also two of the most frequently misused verbs in English as well as mathematical writing.

Wrong	Correct	Comments
1. ***It is*** a rational number between 1 and 2. 2. ***There has*** a rational number between 1 and 2.	***There is*** a rational number between 1 and 2.	The phrase 'there is' is usually used to introduce new information (which does not refer back to previous information); when we say 'it is' there should be something (in this example a rational number) introduced before. 'There has' is a grammatical mistake; always use 'there is/are' (the verb-to-be).
A real number x whose square is negative ***is not exist***.	A real number x whose square is negative ***does not exist***.	Use the auxiliary verb 'do' to form the negative.
This sequence ***is not converge***.	1. This sequence ***does not converge***. 2. This sequence ***is not convergent***.	1. Use the auxiliary verb 'do' to form the negative. 2. Use the adjective form (convergent) when using the verb-to-be.

3.1.5 Converting between different word forms

Most words can be changed slightly to convert from one word form to another, such as *converge* (verb) and *convergent* (adjective). You have to be aware of the differences between the different word forms.

Wrong	Correct	Comments
1. ***Besides from*** completing the square, we can use differentiation to find the minimum value of x^2-4x+8. 2. ***Beside*** completing the square, we can use differentiation to find the minimum value of x^2-4x+8.	***Besides*** completing the square, we can use differentiation to find the minimum value of x^2-4x+8.	In this sentence 'besides' is used in a dependent clause, and should be followed by the -ing form of the verb.
1. The total ***increasement*** in surface area is 6 cm^2. 2. The surface area ***increases*** 6 cm^2.	1. The total ***increase*** in surface area is 6 cm^2. 2. The surface area ***increases by*** 6 cm^2.	The noun form and the verb form are the same ('increase'), albeit pronounced differently. The word 'increasement' does not exist. But in the verb form 'increase' is followed by the dependent preposition 'by', i.e. 'increase by an amount'.
The ***maximal*** possible value of $f(x)$ is 3.	The ***maximum/largest*** possible value of $f(x)$ is 3.	The adjective form of the noun 'maximum' is the same (maximum). In this example the superlative 'largest' can also be used'. 'Maximal' has a slightly different meaning in mathematics.
The ***slanted*** height of the cone is 10 cm.	The ***slant*** height of the cone is 10 cm.	'Slant height' is a compound noun (two nouns combined) like 'traffic light'.
The function $y=x^2$ ***concaves*** upward.	The function $y=x^2$ ***is concave*** upward.	'Concave' is an adjective rather than a verb.
For every rational number x, ***can be*** written as $x=\frac{p}{q}$.	***Every*** rational number x ***can be*** written as $x=\frac{p}{q}$.	When using this phrase only use one clause. The phrase 'every rational number x' is the subject of this clause.
One plus one ***equals to*** two.	1. One plus one ***equals*** two. 2. One plus one ***is equal to*** two.	This is a confusion between the verb form and adjective form. The verb form does not have a dependent preposition.

Wrong	Correct	Comments
1. The ***followings*** are equivalent. 2. The possible values of x are ***as follow***: 3. The result ***is followed***.	1. The ***following statements*** are equivalent. 2. The possible values of x are ***as follows***: 3. The result ***follows***.	'The following' is used in phrases such as 'we have the following'. 'As follows' is used in phrases such as 'the properties/values/results are as follows'. In the incorrect example 'follow' is used in passive voice which means followed by something. In the correct examples 'follow' is used in the active voice because after this expression comes the evidence for the result(s).

3.2 Lexical Errors

A lexical error is making the wrong choice of word for the stylistic context. The sentence is not necessarily grammatically wrong, but does not mean exactly what the author wants to convey.

3.2.1 The order of words

The wordings in the following examples can be slightly revised so that it reads more smoothly.

Wrong	Correct	Comments
1. Let x ***is*** non-negative. 2. Let ***a prime number be x***. 3. Let ***the width of the rectangle be y***.	1. Let x ***be*** non-negative. 2. Let ***x be a prime number***. 3. Let ***y be the width of the rectangle***.	'Let' is used with a bare infinitive form of a verb in the first example. In the second and third examples the convention is 'let + x, y, etc + be + the value which is assigned to x, y, etc' .
Since the divisor $(x-1)(x-2)$ is of degree 2, ***1 is the maximum degree of the remainder***.	Since the divisor $(x-1)(x-2)$ is of degree 2, ***the maximum degree of the remainder is 1***.	The description of the value (the terminology) goes first in a clause, the value/figure goes last.
The smallest value of x^2 ***possible*** is 0.	The smallest ***possible*** value of x^2 is 0.	The word order is 'smallest/largest + possible + value'.
Let x be ***a real positive number***.	Let x be ***a positive real number***.	The word order should be : 'positive/negative + real + number'.

Wrong	Correct	Comments
For all even numbers N, ***we will show that*** $3+N$ is odd.	***We will show that*** $3+N$ is odd for every even number N.	The phrase 'we will show that' usually begins a sentence, although the other way round is not wrong.
If f is a ***strictly continuous increasing*** function, ...	A ***strictly increasing continuous*** function, ...	Keep the words 'strictly' and 'increasing' together when describing a function.

3.2.2 Choice of words

A common type of problems in mathematical writing is the use of wrong words. In this part we look at some general examples of such in English. The wrong usage of mathematical terminology in a particular subject area will be dealt with in Section 5.

Wrong	Correct	Comments
A prime number is a natural number that has no positive factors other than 1 and itself. ***For instance***, if k is a natural number and $k = pq$, where both $p, q > 1$, then k is not a prime number.	A prime number is a natural number that has no positive factors other than 1 and itself (***for instance***, 2 is a prime number). ***In other words***, if k is a natural number and $k = pq$ with $p, q > 1$, then k is not a prime.	'In other words' is used to introduce an explanation or clarification of an idea or concept. 'For instance' is used to give an example of the idea or concept.
1. Therefore x is rational, ***e.g.*** $x = \frac{p}{q}$ for some integers p and q. 2. Let $\lceil x \rceil$ be the least integer greater than or equal to x, ***i.e.*** $\lceil \pi \rceil = 4$.	1. Therefore x is rational, ***i.e.*** $x = \frac{p}{q}$ for some integers p and q. 2. Let $\lceil x \rceil$ be the least integer greater than or equal to x, ***e.g.*** $\lceil \pi \rceil = 4$.	'e.g.' (from the Latin phrase *exempli gratia*) means 'for example'; whereas 'i.e.' (from the Latin phrase *id est*) means 'that is'. The former uses an example to illustrate a concept, while the latter gives an alternative explanation.

3.2.3 Other lexical errors

There are often more than one grammatically correct way to express something, but sometimes some choices of wordsings are more suitable than the others.

Wrong	Correct	Comments
Assume ***the contrary*** that not all of them are zeros.	1. Assume ***on the contrary*** that not all of them are zeros. 2. Assume ***the contrary, i.e.*** not all of them are zeros.	'On the contrary' is used as an adverb phrase with the verb assume. On the other hand, 'the contrary' is a noun and the object of the verb 'assume'. In this case add another clause which gives information about the assumption.
Let m and n be ***odd and even***.	1. Let m and n be ***odd and even respectively***. 2. Let ***m be odd and n be even***.	The wrong example is ambiguous because it does not precisely state which is odd and which is even. The two corrected examples show two possible variations which are much clearer.
This equation has ***finite*** solutions.	1. This equation has ***finitely many*** solutions. 2. This equation has ***a finite number of*** solutions.	Here 'finite' is used to describe the *number* of solutions, rather than the solutions themselves.
The equation $x+3=2x+4-x-1$ has ***infinite*** solutions.	The equation $x+3=2x+4-x-1$ has ***infinitely many*** solutions.	Again, 'infinite' refers to the number of solutions rather than the solutions themselves.

3.3 Other Issues

In this section we collect some other miscellaneous issues as well as some conventional issues. In English language, conventions are a courtesy to the reader, making writing easier to read by putting it in a form that the reader expects and is comfortable with. It includes things such as sentence formations (e.g. complete sentences, punctuation) and conventions of print (e.g. spelling, capitalisation). Many of these do not exist in oral language, so you have to consciously learn them in written language.

3.3.1 Words with similar pronunciations

Some words sound the same (or similar) when pronounced, but are in fact spelt differently, and may have different meanings.

Wrong	Correct	Comments
The number x ***can not*** be rational.	The number x ***cannot*** be rational.	'Cannot' and 'can not' have different meanings. For example 'he cannot do it' means he does not have the ability to do it, while 'he can not do it' usually implies he has the ability to do it but also has the option of not doing it.
Without ***lost*** of generality, we have...	Without ***loss*** of generality, we have...	The noun form 'loss' is used in this context.
1. This ***proofs*** that $x > 0$. 2. This completes the ***prove***.	1. This ***proves*** that $x > 0$. 2. This completes the ***proof***.	'Proof' is a noun whereas 'prove' is the corresponding verb.

3.3.2 Frequently confused words

Some words are often misused in mathematical writing.

Wrong	Correct	Comments
Rotate $\triangle ABC$ ***clockwisely*** by $90°$.	Rotate $\triangle ABC$ ***clockwise*** by $90°$.	'Clockwise' is already an adverb. The word 'clockwisely' does not exist.
The numbers a, b and c are ***pairwisely*** different.	The numbers a, b and c are ***pairwise*** different.	Again, the word 'pairwisely' does not exist.
Here is a ***counter example***.	Here is a ***counterexample***.	Here 'counter' is not a word, but rather a prefix attached to 'example' meaning 'the opposite'.
Let x be a ***non zero*** number.	Let x be a ***nonzero*** number.	Again, 'non' is not a word, but rather a prefix attached to 'zero' meaning 'not'.
From this we ***deduct*** that the equation has no solution.	From this we ***deduce*** that the equation has no solution.	Although 'deduction' is the noun for both the verbs 'deduct' and 'deduce', the two verbs have different meanings. 'Deduct' means 'subtract', while 'deduce' means 'draw a logical conclusion'.

3.3.3 Use of linking verbs and punctuation

Connectives are very important in mathematical writing as they show the logical relationship between different sentences. There are some conventions and rules on the use of such words.

Note also that there should only be one verb in a simple sentence. When there is more than one verb, we need a conjunction (e.g. 'and') to properly link the phrases together.

Wrong	Correct	Comments
If $x > 0$**. *Then*** $2x > 0$.	If $x > 0$**, *then*** $2x > 0$.	When using 'if' for assumption, always follow it up with something in the same sentence. 'If $x > 0$' is not a complete sentence.
When $x > 0$**. *Then*** $2x > 0$.	When $x > 0$**,** $2x > 0$.	Same as using 'if', when using 'when' for assumption, always follow it up with something in the same sentence. 'When $x > 0$' is not a complete sentence.
Since x is non-negative**. *We*** have $x + 1 > 0$.	Since x is non-negative**, *we*** have $x + 1 > 0$.	'Since' is used to join dependent clauses to independent clauses and therefore must go in sentences which have two clauses. 'Since x is non-negative' is not a complete sentence.
Let x be non-negative**, *then*** $x + 1 > 0$.	Let x be non-negative**. *Then*** $x + 1 > 0$.	When using 'let' and 'then' to list steps of proofs use two sentences, one for each step.
Suppose x is non-negative**, *then*** have $x + 1 > 0$.	Suppose x is non-negative**. *Then*** have $x + 1 > 0$.	When using 'suppose' and 'then' to list steps of proofs use two sentences.

3.3.4 Spelling

Make sure that you spell the words correctly. Apart from those mentioned in Sections 3.3.1 and 3.3.2, some words tend to be misspelt a lot. Surprisingly, it is not uncommon for students to misspell *true* as *ture* and *false* as *flase*, two words which occur a lot in mathematical writing.

4 The Use of Symbols

Equations are essentially made up from symbols – numbers, equality sign, variables and so on. There are many other symbols in mathematics as well. It is important that symbols be used properly, for otherwise the resulting sentence may deviate from the intended meaning.

4.1 The Symbols '=' and '≠'

The symbol '=' is probably one of the very first mathematical symbols one learns. It means the expressions on its two sides are the same. This symbol is also one of the most misused symbols, as people abuse it in various situations. The symbol '≠', on the other hand, means the two sides are not equal, and we have to be careful about its usage too.

4.1.1 Are they equal?

When using the equal sign, make sure that the expressions on the two sides are indeed equal.

Wrong	Correct	Comments
By first principles, the derivative of x^2 is $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\frac{(x^2+2xh+h^2)-x^2}{h}$ $=2x$	By first principles, the derivative of x^2 is $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{(x^2+2xh+h^2)-x^2}{h}$ $=2x$	The answer is correct but it is wrong to omit the limits in the working out. As the argument stands Line 1 and Line 2 in the wrong example are surely not equal (neither do Line 2 and Line 3).
To find the third derivative of xe^x, we have $xe^x = xe^x + e^x$ $= xe^x + 2e^x$ $= xe^x + 3e^x$	To find the third derivative of xe^x, we have $\frac{d^3}{dx^3}(xe^x) = \frac{d^2}{dx^2}(xe^x + e^x)$ $= \frac{d}{dx}(xe^x + 2e^x)$ $= xe^x + 3e^x$	In the first (wrong) example, the expressions are clearly not equal.
Since the derivative is $2x$, the slope of the tangent at $(2, 5)$ is 4. Hence the equation of the tangent is $\frac{y-5}{x-2} = 4 = y = 4x - 3.$	Since the derivative is $2x$, the slope of the tangent at $(2, 5)$ is 4. Hence the equation of the tangent is $\frac{y-5}{x-2} = 4,$ *which is the same as* $y = 4x - 3.$	What the wrong example intended to say was that the equation '$\frac{y-5}{x-2} = 4$' is 'equal' to the equation '$y = 4x-3$'. But as it stands it says much more than that — — for example the middle equality reads $4 = y$, which does not make sense.

Wrong	Correct	Comments
We row reduce the matrix to find $$\begin{bmatrix}1 & 0\\2 & 4\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}.$$ Hence it is invertible.	We row reduce the matrix to find $$\begin{bmatrix}1 & 0\\2 & 4\end{bmatrix} \to \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}.$$ Hence it is invertible.	The matrices are row equivalent but not equal (we say two matrices are equal if and only if all their entries are the same). The proper way is to use an arrow; usually we also indicate the operations carried out, for example, $\xrightarrow{-2R_1+R_2}$ means we add -2 times row 1 to row 2.

4.1.2 Non-transitivity of $\neq$

The symbols '$=$' and '$\neq$', like many others, are used to describe the relationship between *two* things. Sometimes more than two things are involved and we may still use these symbols successively, e.g. $1 < 2 < 3$, but only when the symbol is *transitive* — in this example '$<$' is transitive since if $1 < 2$ and $2 < 3$, then we must have $1 < 3$. Likewise, '$=$' is transitive. However, '$\neq$' is not.

Wrong	Correct	Comments
Since $a \neq b \neq c$, we have...	1. Since $a \neq b$, $b \neq c$, **and** $a \neq c$, we have... 2. Since a, b, c **are pairwise distinct**, we have...	The wrong example intended to mean that all three values a, b and c are different but with the way it is written, a and c could be equal (for example consider $1 \neq 2 \neq 1$).

4.1.3 Proper order

As previously mentioned, '$=$' is transitive and so we can equate three or more expressions in a single chain of equalities. However, we have to be careful about the order.

Wrong	Correct	Comments
Since $\frac{x^4y^4}{8} = 2$, we have $\boldsymbol{x^4y^4 = (xy)^4} = 8\times2 = 16$ and so $xy = \sqrt[4]{16} = 2$.	Since $\frac{x^4y^4}{8} = 2$, we have $\boldsymbol{(xy)^4 = x^4y^4} = 8\times2 = 16$ and so $xy = \sqrt[4]{16} = 2$.	When studying a chain of equalities, one naturally tries to figure out why each equality sign holds. In the wrong example, one can understand why $x^4y^4 = (xy)^4$, but then for the second inequality $(xy)^4 = 8\times2$, one gets stuck. In fact it is x^4y^4 that is equal to 8×2, so switching the order makes it much easier to follow.

4.2 The Symbols '$\Rightarrow$' and '$\Leftrightarrow$'

The first symbol means 'implies' and the second symbol means 'is equivalent to' (or 'if and only if'). They are used to relate different statements and are some of the most frequently used symbols. Yet, they are also some of the most commonly misused symbols.

Wrong	Correct	Comments
If $\boldsymbol{x = 1 \Rightarrow}$ $x + 1 = 2$.	1. If $x = 1$**, then** $x + 1 = 2$. 2. $\boldsymbol{x = 1 \Rightarrow}$ $x + 1 = 2$.	The easiest way to see what is wrong is to convert back to English. The wrong example reads 'if x equals 1, implies $x + 1$ equals 2', which clearly does not seem correct. Note that the correct example reads 'x equals 1 implies $x + 1$ equals 2', which is perfectly fine.
Hence we have $x + 1 = 5 \boldsymbol{\Rightarrow} x = 4$.	Hence we have $x + 1 = 5$**,** ***which implies*** $x = 4$.	The problem in the wrong example is that the statement '$x + 1 = 5 \Rightarrow x = 4$' is true regardless to what happens before, contrary to what we expect by the use of the connective 'hence'. The intended meaning was that the previous argument implies that $x + 1 = 5$, which then implies $x = 4$. Note that one may try to interpret the wrong example as $$(\text{Hence we have } x + 1 = 5) \Rightarrow (x = 4)$$ but this is not correct either since 'hence we have $x + 1 = 5$' is not a statement (while '$x + 1 = 5$' is).

Wrong	Correct	Comments
The quadratic function has a critical point at $(2, 3)$. From this we see that the point $(2, 3)$ must be a maximum. $\Rightarrow x = 2$.	The quadratic function has a critical point at $(2, 3)$. From this we see that the point $(2, 3)$ must be a maximum ***and therefore*** $x = 2$.	In general try not to use a symbol in the middle of nowhere. In the wrong example it is unclear which statement implies $x = 2$. Note that '$\Rightarrow x = 2$' is an incomplete sentence and it must be preceded by a statement. While 'the point $(2, 3)$ must be a maximum' is a statement, 'from this we see that the point $(2, 3)$ must be a maximum' is not.

4.3 The Symbols '$\forall$' and '$\exists$'

The symbol '$\forall$' reads 'for all' and the symbol '$\exists$' reads 'there exists' and that is precisely what they mean. To see whether the symbols are used correctly, the easiest way to read the sentence to see if it is a grammatically correct complete sentence and if it makes sense.

Wrong	Correct	Comments
1. Let A be the set of positive odd numbers. Then $2\|a + 1 \; \exists \; a \in A$. 2. If $f(0) < 0$, $f(1) > 0$ and f is continuous, then $\forall \; c \in [0, 1]$ s.t. $f(c) = 0$.	1. Let A be the set of positive odd numbers. Then $2\|a + 1 \; \forall \; a \in A$. 2. If $f(0) < 0$, $f(1) > 0$ and f is continuous, then $\exists \; c \in [0, 1]$ s.t. $f(c) = 0$.	The wrong examples mixed up the meaning of the symbols '$\forall$' and '$\exists$'.
$f(c) = 0 \; \exists \; c \in [0, 1]$	1. **$\exists$ $c \in [0, 1]$ s.t.** $f(c) = 0$ 2. $f(c) = 0$ ***for some*** $c \in [0, 1]$	'$\exists$' means 'there exists' rather than 'for some'. Note there is no proper symbol for 'for some'.
$\exists \; x \in \mathbb{R} \; \forall \; x > 1000$	$\exists \; x \in \mathbb{R}$ ***s.t.*** $x > 1000$	'$\forall$' means 'for all', not 'such that'.
Since $\max S = 1$, we have $x \leq 1$ ***for*** $\forall \; x \in A$.	Since $\max S = 1$, we have $x \leq 1 \; \forall \; x \in A$.	'$\forall$' reads 'for all', so 'for $\forall$' would read 'for for all' which is wrong.

Wrong	Correct	Comments
$\forall\ x \in \mathbb{R}$ ***s.t.*** $x^2 \geq 0$	$x^2 \geq 0\ \forall\ x \in \mathbb{R}$	'$\forall\ x \in \mathbb{R}$ s.t. $x^2 \geq 0$' is not even a complete sentence (try to read it). When 'such that' follows 'for all', we do not really mean 'for all', bur rather 'for those which satisfy the subsequent condition'. For example, 'for all positive even integers n such that $n > 6$, we can write n as the sum of two odd primes' — here we do not really mean 'for all positive even integers n', but only those which satisfy the subsequent condition described after 'such that', i.e. $n > 6$.

4.4 The Symbols '$\in$' and '$\subseteq$'

The first symbol is set membership and simply reads 'in' or 'belongs to' and we use it to denote that a certain element lies in a set, for example $2 \in \mathbb{N}$ or $\pi \in \mathbb{R}$. The second one means 'subset' and we use it to denote that a collection of elements all lie in a certain set, for example $\mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$.

Wrong	Correct	Comments
1. $(0,1) \in \mathbb{R}$ 2. If $x = 5$, then $x \subseteq \mathbb{R}$.	1. $(0,1) \subseteq \mathbb{R}$ 2. If $x = 5$, then $x \in \mathbb{R}$.	An interval is a subset of $\mathbb{R}$, and so the subset symbol '$\subseteq$' should be used. In the second example x is a real number, so the set membership symbol '$\in$' should be used.

4.5 Overusing Symbols

We conclude this section with a warning on using symbols. While using symbols correctly is essential for presenting mathematics accurately and concisely as we have shown throughout this section, there is always a danger of overusing them. For example, consider the following definition of a function $f(x)$ being continuous at the point a:

- $\forall \varepsilon > 0\ \exists \delta > 0$ s.t. $|x - a| < \delta \Rightarrow |f(x) - f(a)| < \varepsilon$.

While it is perfectly sound and correct, it is a bit difficult to read. Reducing the use of symbols would give

- For all $\varepsilon > 0$, there exists $\delta > 0$ such that $|f(x) - f(a)| < \varepsilon$ whenever $|x - a| < \delta$.

In fact, in professional mathematical writing, symbols are usually kept to a minimum except when discussing logic. Of course sometimes we may want to use symbols to save time (e.g. in exam situations). The bottom line is that they are used correctly and form part of complete sentences, and that the whole argument is reasonably readable.

5 The Use of Terminology

The principle of using mathematical terminology is basically choosing the right word for the right occasion. Here we split the examples according to the subject area.

5.1 Elementary Algebra

Here we refer to things such as functions, polynomials and so on (as opposed to abstract algebra), which most secondary school students should know.

Wrong	Correct	Comments
1. Suppose $p(x) = ax + b$, where a and b are ***constant***. 2. Hence $p(x)$ and $q(x)$ are ***constants*** polynomials.	1. Suppose $p(x) = ax + b$, where a and b are ***constants***. 2. Hence $p(x)$ and $q(x)$ are ***constant*** polynomials.	The word 'constant' can be used as an adjective or a noun. As an adjective, it is used to describe the non-varying property of functions. As a noun, it is used to refer to a fixed and well-defined number and has a plural form 'constants'.
Since $6 = 2 \times 3$, so 6 ***divides*** 3.	1. Since $6 = 2 \times 3$, so 6 ***is divisible by*** 3. 2. Since $6 = 2 \times 3$, so 3 ***divides*** 6.	A rule of thumb is that if a divides b (where a, b are positive integers), then a must be the smaller number.
We can ***move*** the graph of $y = e^x$ to the left by 1 unit to obtain the graph of $y = e^{x+1}$.	We can ***translate*** the graph of $y = e^x$ to the left by 1 unit to obtain the graph of $y = e^{x+1}$.	When referring to moving graphs by a fixed vector the verb 'translate' is used.
The ***multiplication*** of two positive numbers is positive.	The ***product*** of two positive numbers is positive.	The result of multiplication is 'product'.
We count the number of integers between 1 and 100 which are not ***prime numbers***.	We count the number of integers between 1 and 100 which are not ***prime***.	There is redundant information in the wrong example ('integers' are 'numbers'). The convention is to use only the adjective.

Wrong	Correct	Comments
The ***thousand*** digit of 12345 is 2.	The ***thousands*** digit of 12345 is 2.	When referring to digits use the terms 'unit digit', 'tens digit', 'hundreds digit', etc.
The ***tenth*** digit of 12.34 is 1.	The ***tens*** digit of 12.34 is 1.	The tenth digit refers to the digit immediately to the right of the decimal point, i.e. 3 in this example.
1. N is divisible by 7 ***when*** N^2 is divisible by 7. 2. Let $f(x) = 2x$. ***If*** $x = 3$, $f(x) = 6$.	1. N is divisible by 7 ***if*** N^2 is divisible by 7. 2. Let $f(x) = 2x$. ***When*** $x = 3$, $f(x) = 6$.	In mathematical proofs the convention is to use 'if' rather than 'when' for describing a condition. But in the second example, it is more popular to use 'when' since we are talking about a variable taking on a certain value.

5.2 Geometry

Plane geometry is another popular topic in the mathematics syllabus of secondary school.

Wrong	Correct	Comments
Denote the ***centre of the circumcentre*** of ABC by O.	Denote the ***circumcentre*** of ABC by O.	'Circumcentre' is the centre of the circumscribed circle.
1. The quadrilateral $ABCD$ is ***concyclic***. 2. The points A, B, C and D are ***cyclic***.	1. The quadrilateral $ABCD$ is ***cyclic***. 2. The points A, B, C and D are ***concyclic***.	'Concyclic' is used to describe points whereas 'cyclic' is used to describe polygons.
Let $ABCD$ ***form*** a square.	1. Let $ABCD$ ***be*** a square. 2. The points A, B, C and D ***form*** a square.	The correct examples are the mathematical conventions.
The length of the ***perimeter*** of the circle is 4π cm.	1. The length of the ***circumference*** of the circle is 4π cm. 2. The ***perimeter*** of the circle is 4π cm.	In mathematics 'circumference' is the outer boundary of a circle, while 'perimeter' means 'length of the boundary'.

5.3 Mathematical Induction

Mathematical induction is a widely used technique of proof in many branches of mathematics. Study the following proof. Can you point out all the problems in it?

<u>Question</u>

Show that $1+2+\cdots+n=\dfrac{n(n+1)}{2}$ for all positive integers n.

<u>Solution</u>

Let $S(n)$ be the statement $1+2+\cdots+n=\dfrac{n(n+1)}{2}$ for all positive integers n.

- $S(1)$ is true since

$$\begin{aligned}1+2+\cdots+1&=\frac{1(1+1)}{2}\\1&=1\end{aligned}$$

- Assume $S(k)$ is true for all positive integers k, s.t. $1+2+\cdots+k=\dfrac{k(k+1)}{2}$.
- When $S(k+1)$, we have

$$\begin{aligned}S(k+1)&=1+2+\cdots+(k+1)\\&=\frac{k(k+1)}{2}+(k+1)\\&=\frac{k(k+1)+2(k+1)}{2}\\&=\frac{(k+1)(k+2)}{2}\\&=\frac{(k+1)[(k+1)+1]}{2}\end{aligned}$$

Hence $n=k+1$ is also true.

By the principal of mathematical induction, $S(n)$ is true for all positive integers n.

There are close to ten errors in the proof. How many can you find?

Wrong	Correct	Comments
Let $S(n)$ be the statement $\mathbf{1+2+\cdots+n=\dfrac{n(n+1)}{2}}$ **for all positive integers n.**	Let $S(n)$ be the statement $\mathbf{1+2+\cdots+n=\dfrac{n(n+1)}{2}}$.	$S(n)$ is a statement that depends on the value of n. We want to prove that $S(n)$ is true for all n.
$\mathbf{1+2+\cdots+1}$	$\mathbf{1}$	We start adding from 1 and end at n. When $n=1$, there is only one term and we should not write '+2' at all.

Wrong	Correct	Comments
$\mathbf{1+2+\cdots+1=\frac{1(1+1)}{2}}$ $\mathbf{1=1}$	$\mathbf{LHS=1}$ $\mathbf{RHS=\frac{1(1+1)}{2}=1}$	When proving an equality, we do not simplify both sides simultaneously. We may either start from one side and reach the other, or simplify both sides separately to obtain the same value or expression.
Assume $S(k)$ is true for ***all positive integers*** k.	Assume $S(k)$ is true for ***some positive integer*** k.	'$S(k)$ is true for all k' is precisely the statement we need to prove, and therefore it does not make sense to assume $S(k)$ is true for all k.
s.t. $1+2+\cdots+k=\frac{k(k+1)}{2}$.	***i.e.*** $1+2+\cdots+k=\frac{k(k+1)}{2}$.	Here we want to use the phrase 'that is' rather than 'such that', because we want to explain what we mean by $S(k)$ is true, rather than to talk about some consequences.
When $\boldsymbol{S(k+1)}$	When $\boldsymbol{n=k+1}$	When we say 'when $n=\cdots$', we are discussing what happens under that particular value of n. Here we need to prove that $S(k+1)$ is true, so we cannot say 'when $S(k+1)$'.
$\boldsymbol{S(k+1)=1+2+\cdots+(k+1)}$	$\boldsymbol{\mathrm{LHS}=1+2+\cdots+(k+1)}$	$S(k+1)$ is a statement and cannot be equal to $1+2+\cdots+(k+1)$.
Hence $\boldsymbol{n=k+1}$ is also true.	Hence $\boldsymbol{S(k+1)}$ is also true.	Only a statement can be true or false. $S(k+1)$ is a statement, but $n=k+1$ is not a statement since n and k are dummy variables and we cannot say $n=k+1$ is true or false.
By the ***principal*** of mathematical induction, $P(n)$ is true for all $n\geq 2$.	By the ***principle*** of mathematical induction, $P(n)$ is true for all $n\geq 2$.	'Principal' is the head of a school and 'principle' is a general law or primary truth.

5.4 Functions and Calculus

These errors are specific to the use of terminology in the concepts of functions, limits, differentiation and integration.

Wrong	Correct	Comments
-1 is not ***the domain*** of $\sqrt{x}$.	1. -1 is not ***in the domain*** of $\sqrt{x}$. 2. -1 is not ***an element of the domain*** of $\sqrt{x}$.	'Domain' is a set and the intended meaning here is to clarify whether -1 is an element of this set.
The ***differentiation*** of x^2 is $2x$.	The ***derivative*** of x^2 is $2x$.	'Differentiation' is the process of finding the derivative.
To solve for the points of intersection of the graphs, we ***suppose*** $x+1=2x+3$.	To solve for the points of intersection of the graphs, we ***set*** $x+1=2x+3$.	'Suppose' is used for assumption. In this example there is no assumption; rather we assign two expressions to be equal to find x.
The function $f(x) = 3x$ is strictly increasing ***in*** the interval $[0,1]$.	The function $f(x) = 3x$ is strictly increasing ***on*** the interval $[0,1]$.	It is a mathematical convention to say 'on an interval' rather than 'in an interval'.
Let $f(x)=\sin x$. When $\boldsymbol{f(0)}$, we have $\boldsymbol{\sin 0 = 0}$.	Let $f(x) = \sin x$. When $\boldsymbol{x=0}$, we have $\boldsymbol{f(0) = \sin 0 = 0}$.	The statement $\sin 0 = 0$ is true regardless of what happens to $f(0)$. Also, there must be a condition following the word 'when'. For example, you could say 'when x is 2, the value of $f(x)$ is 5'. But in the example, $f(0)$ is not a condition.
Note that $\ln(\sin 0)$ ***has no solution***.	Note that $\ln(\sin 0)$ is ***undefined***.	We can only say that an equation (with some sort of variable) has no solution; but here $\ln(\sin 0)$ is a value (although undefined).
The $(k+1)$-th derivative of $\sin x$ is $\frac{d^k d}{dx^k x}(\sin x) = \frac{d^k}{dx^k}(\cos x)$.	The $(k+1)$-th derivative of $\sin x$ is $\frac{d^k}{dx^k}\left(\frac{d}{dx}(\sin x)\right) = \frac{d^k}{dx^k}(\cos x)$.	The notation in the first example is wrong.

5.5 Linear Algebra

These errors are specific to the use of terminology in the theory of matrices and vector spaces.

Wrong	Correct	Comments
The ***second*** entry of A is 2.	The ***(1,2)-entry*** of A is 2.	It is unclear what the 'second' entry of a matrix is — whether it is the second element in the first row or the first element in the second row.
Since A is 3×4 and B is 4×5, the matrix $\boldsymbol{A \times B}$ is 3×5.	Since A is 3×4 and B is 4×5, the matrix $\boldsymbol{AB}$ is 3×5.	Although not wrong, it is not a common practice to denote the multiplication of matrices using the '$\times$' sign. Most of the time no symbol is used at all.
The ***matrix*** A ***is*** positive.	The ***entries of the matrix*** A ***are*** positive.	'Positive' is used to describe real numbers.
For the $\boldsymbol{(2,3)}$***-entry of*** $\boldsymbol{A}$ $= 2 + 3 = 5$.	1. ***The*** $\boldsymbol{(2,3)}$***-entry of*** $\boldsymbol{A}$ $= 2 + 3 = 5$. 2. For the $(2,3)$-entry of A, ***it is equal to*** $2 + 3 = 5$.	'The $(2,3)$-entry of A' can be equal to a number, but 'for the $(2,3)$-entry of A' is a phrase and cannot be equal to a number.
When E is ***a Type III***, ...	1. When E is ***of Type III***, ... 2. When E is ***a Type III elementary matrix***, ...	'Type III' is an adjective, so it should be followed by a noun, or we add the preposition 'of' beforehand.
If A ***is rank*** 3, then it must be invertible.	1. If A ***has rank*** 3, then it must be invertible. 2. If A ***is of rank*** 3, then it must be invertible. 3. If ***rank*** $A = 3$, then it must be invertible.	'A is rank 3' is not a proper complete sentence.
We ***solve*** the characteristic ***polynomial*** of A as follows.	1. We ***find*** the characteristic polynomial of A as follows. 2. We solve the characteristic ***equation*** of A as follows.	We can 'solve' an equation but not a polynomial.

Wrong	Correct	Comments
The matrix A is linearly independent and so must be invertible.	The ***rows*** of the matrix A are linearly independent and so must be invertible.	Linear independence refers to elements of a vector space (e.g. row/column vectors). The intended meaning here is that the row vectors of A (multiple objects) are linearly independent, not the matrix A itself (a single object) being linearly independent.
Hence $\mathbf{x}, \mathbf{y}$ is a basis.	Hence $\{\mathbf{x}, \mathbf{y}\}$ is a basis.	A basis is a set of vectors in a vector space and therefore must be expressed as a set.
Since A has rank 2, we have $\mathbf{Null}\, A = 3$.	Since A has rank 2, we have $\mathbf{dim(Null}\, A) = 3$.	Null A refers to the null space of A and cannot be equal to a number. What is equal to 3 is the dimension of its null space (also called its nullity).
Since A has rank 2, the ***rank*** of its null space is 3.	Since A has rank 2, the ***dimension*** of its null space is 3.	'Null space' is not a matrix and hence has no rank.
If a 3×3 matrix is invertible, its ***number of rank*** is 3.	If a 3×3 matrix is invertible, its ***rank*** is 3.	'Rank' is already a number.

6 Miscellaneous

6.1 Handwriting

It is important to write in a neat and legible manner. The following lists some frequently confused characters:

- 't' vs '$+$'
- '1' vs 'l' vs 'I' (in particular the natural logarithm is 'ln', not 'In'!)
- 'x' vs '$\times$'
- 'p' vs 'ρ'
- 'a' vs 'α' vs '2'
- '0' vs '6' vs 'σ'

On a side note, Greek letters are extensively used by mathematicians (26 English letters are often insufficient!). One should learn all the Greek letters in order to master the mathematical language.

6.2 Presentation

Always write in clear order (avoid writing in 'two columns' on the same page as it usually hinders reading) and cross out unwanted materials (cross out those and only those words which you don't want; crossing out one word more or one word fewer can lead to a totally different meaning). Highlighting the final answer sometimes also helps make the overall presentation neater.

Also, there are certain expressions such as $\frac{1}{2x}$ and $\frac{1}{2}x$ which you need to distinguish carefully. Sometimes it also helps by writing with proper indentation. For example,

> .... There are two cases:
>
> Case 1: ....
> ....
>
> Case 2: ....
> ....
>
> Combining the two cases, ...

is more readable than

> .... There are two cases. For the first case, ... For the second case, ... Combining the two cases, ...

6.3 Avoiding Isolated Equations

We began this writing guide by saying that complete sentences should be used in mathematical writing. We conclude it by saying that complete *paragraphs* should be used. In short, this means we should avoid writing a few isolated equations without explaining the logical relationship between them. For example, consider the following:

$$5x + 1 = 16$$
$$5x = 15$$
$$x = 3$$

This is probably how primary and secondary school students 'write mathematics'. Indeed these can be considered as complete sentences. (Try to read them: '5 times x plus 1 is equal to 16. 5 times x is equal to 15. x is equal to 3.' Three complete sentences!) However when put together this makes no sense — what are we trying to say? Are we *assuming* the first equation holds? Or is it true that the first equation *does* hold because of some reason? Furthermore, what is the relationship between these equations? These are important issues and must be clarified by using proper connectives or symbols. For instance, two possible ways to connect these equations together are as follows:

1. According to the question, we have $5x + 1 = 16$. Since $5x + 1 = 16 \Leftrightarrow 5x = 15 \Leftrightarrow x = 3$, we conclude that the value of x is 3.

2. It follows from our previous discussion that $5x + 1 = 16$. This is equivalent to $5x + 1 = 15$, or $x = 3$.

If you are still not convinced of the importance of avoiding isolated equations, consider the following (wrong) demonstration:

$$\begin{aligned}\sqrt{x-2} &= x - 4\\ x - 2 &= x^2 - 8x + 16\\ x^2 - 9x + 18 &= 0\\ (x-3)(x-6) &= 0\\ x &= 3 \text{ or } 6\end{aligned}$$

Yet if we plug $x = 3$ into the original equation, the two sides are not equal. Some would argue that *since we have squared both sides*, it is necessary that we check the 'solutions' obtained at the end. However this explanation is neither complete nor convincing — apart from naturally asking why (squaring both sides would matter), you can easily find many other examples in which you would end up with such 'wrong solutions' even though you haven't squared both sides in the process. Ultimately, it is the relationship between different equations that matters.

The above can be rewritten as follows:

- To solve the equation $\sqrt{x-2} = x - 4$, we note that

$$\begin{aligned}\sqrt{x-2} = x - 4 &\implies x - 2 = x^2 - 8x + 16\\ &\implies x^2 - 9x + 18 = 0\\ &\implies (x-3)(x-6) = 0\end{aligned}$$

  Hence the only possible values of x are 3 and 6. When $x = 3$, the left hand side of the equation is 1 while the right hand side is -1; when $x = 6$ both sides are equal to 2. Thus we conclude that $x = 6$ is the only solution.

As the last piece of advice, try to learn mathematical writing by reading how professional mathematicians write. Pay special attention to how words and symbols are integrated to form complete sentences, and how complete sentences are linked together by connectives to form coherent paragraphs that present rigorous mathematical arguments.