

# Polynomial families and Waring's problem

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## Abstract

Let  $A$  be a commutative ring. A subset  $X$  of  $A^n$  is a polynomial family with  $d$  parameters if it is the range of a polynomial map from  $A^d$  to  $A^n$ . It is an old question of Skolem (1938) whether the group  $\mathrm{SL}_2(A)$  with  $A$  being the set of integers is a polynomial family. Only recently, Vaserstein (2010) answered Skolem's question in the affirmative. For the first part of the talk, I will discuss my result of Skolem's conjecture in the function field case  $\mathrm{SL}_2(A)$ , where  $A$  is the polynomial ring over a finite field of  $q$  elements.

The Waring problem asks whether for each positive integer  $n$ , every nonnegative integer is the sum of a bounded number of  $n$ th powers. Equivalently, it asks whether for each positive integer  $n$ , the set of nonnegative integers is a polynomial family with a bounded number of parameters, say  $d$  for the polynomial map from  $Z^d$  to  $Z$  defined by sending  $(x_1, \dots, x_d)$  to  $x_1^n + \dots + x_d^n$ . The second part of the talk is a joint work with Michael Larsen (Indiana University) on the Waring problem for unipotent algebraic groups.