Gaussian Channels with Feedback

Shunsuke Ihara
Nagoya University

Abstract: We consider discrete-time additive Gaussian noise channels with feedback. We assume that the noises are zero mean stationary Gaussian processes and that average power constraints are imposed on the channels. We are interesting the problems concerning

(1) Feedback capacity

(2) Optimal coding schemes

(3) Asymptotic behavior of error probability.

As is well known, if a channel is without feedback, we have the formula for the capacity. It is also well known that if the Gaussian noise is white, namely stationary and memoryless, the capacity is not increased with feedback. On the other hand, if the Gaussian channels are with feedback, despite numerous lower and upper bounds have been reported, closed-forms for the feedback capacities are not known, except some special cases.

In the case where the additive Gaussian noise is a first-order moving average (MA(1)) process, Kim (2006) obtained a closed-form of the feedback capacity. He showed that the capacity is achieved by sending a Gaussian message with help of linear feedback. The coding scheme may be called a SK (Schalkwijk-Kailath) scheme. Moreover, Kim (2010) gave a characterization of the feedback capacity of the stationary Gaussian channels as the solution to a variational problem in the spectral density function of the Gaussian noise.

The main aim of this talk is to give a new proof of the optimality of the SK scheme for the Gaussian MA(1) noise channel.

We also discuss on the decoding error probability. Let us consider an additive Gaussian noise channel with feedback. Let C be the maximum mutual information one can transmit by using the SK scheme. Then we can show that, if the rate of the message is less than C, the minimum decoding error probability converges to zero exponentially fast, and that the order of exponential is linearly increasing with block length. The similar asymptotic behavior of the minimum error probability can be shown for continuous-time additive Gaussian noise channels with feedback.