

Workshop on Complex Geometry
October 24 - 27, 2023

Titles and Abstracts

Ya Deng CNRS

Constructing Shafarevich morphism for representations in positive characteristic and hyperbolicity

In 1972, Shafarevich asked the question of whether the universal covering of a complex projective variety is homomorphically convex, now known as the Shafarevich conjecture. In cases where fundamental groups are complex linear, this conjecture has been proven by Eyssidieux et. al in 2010. In this talk, I will consider any quasi-projective manifold X and any linear representation $\rho : \pi_1(X) \rightarrow GL_N(K)$ with K any field of positive characteristic. I will explain

1. the construction of the Shafarevich morphism for ρ ;
2. the proof of the strong Green-Griffiths-Lang conjecture when ρ is big.

I will also discuss some applications, including the proofs of Claudon-Höring-Kollár's conjecture and Campana's abelianity conjecture for projective varieties with linear fundamental groups in any characteristic. This is jointly with Katsutoshi Yamanoi.

Lawrence Ein University of Illinois at Chicago

Saturation bounds for a smooth projective variety

Let S be the complex polynomial ring of $r + 1$ variables. Suppose that f_0, \dots, f_p are in S with $\deg f_i = d_i$ and assume that $d_0 \geq d_1 \dots \geq d_p$. Suppose that X the closed subscheme in \mathbb{P}^r defined by J is a smooth projective variety of dimension n . Then $J \subset I_X$, the homogenous ideal of X and I_X/J is of finite length. So $(I_X/J)_t = 0$ for all $t \gg 0$. The least such integer t is called the saturation degree of J . Bombieri asked the question whether one can bound the saturation degree of J in terms of the d_i 's. In this talk, we'll discuss joint work with Ha and Lazarsfeld on answering this question.

Philippe Eyssidieux Université Grenoble Alpes

L^2 Betti numbers of complex algebraic manifolds and of Hodge modules

I will outline my work, partially in progress and partially in collaboration with Bastien Jean, on L^2 constructible cohomology, Mixed Hodge structures on the reduced L^2 cohomology of a Mixed Hodge Modules.

Samuel Grushevsky Stony Brook University

Ends of strata of differentials

The moduli spaces of complex curves together with a meromorphic differential with prescribed multiplicities of zeros and poles are the phase spaces of the action of $GL(2, R)$ in Teichmuller dynamics, and are natural interesting geometric subvarieties of the moduli of curves with marked points. The geometry and topology of the strata remain mysterious, and we will present perhaps the first topological result beyond the number of connected components: that every connected component of every stratum in genus at least 2 has only one end; equivalently, this says that the boundary of a smooth compactification is connected. Based on joint work with Ben Dozier.

Gordon Heier University of Houston

A Schmidt-Nochka theorem for closed subschemes in subgeneral position

The seminal Subspace Theorem due to Schmidt has been generalized in many ways over the years. One such generalization was our formulation in terms of Seshadri constants for closed subschemes in general position. In this talk, we will present a further refinement in terms of Seshadri constants and weighted sums involving closed subschemes in subgeneral position. The use of weights in this context is motivated by Nochka's proof of a conjecture of Cartan on defects of holomorphic curves in projective space relative to a possibly degenerate set of hyperplanes. Our refinement yields new Diophantine inequalities, including an extension of inequalities of Nochka and Ru-Wong from hyperplanes in m -subgeneral position to hypersurfaces in m -subgeneral position in projective space, proving a sharp result in dimensions 2 and 3, and coming within a factor of $3/2$ of a sharp inequality in all dimensions. This is joint work with Aaron Levin.

Pak-Tung Ho Tamkang University

Ancient caloric functions on pseudohermitian manifolds

For any Riemannian manifold with polynomial volume growth, Colding and Minicozzi obtained a sharp bound on the dimension of the space of ancient caloric functions with polynomial growth. In this talk, we explain how we obtain a sharp bound on the dimension of the space of ancient pseudohermitian caloric functions with polynomial growth on a pseudohermitian manifold satisfies doubling volume property and parabolic mean value property.

Jun-Muk Hwang Institute for Basic Science

Minimal rational curves whose VMRT at a general point is an adjoint variety

For a family of minimal rational curves on a uniruled projective manifold, its VMRT at a point is the projective subvariety consisting of tangent directions of minimal rational curves through that point. In a joint work with Qifeng Li, we study families of minimal rational curves whose VMRT at a general point is the adjoint variety of a simple Lie algebra. Nontrivial examples arise from wonderful group compactifications and hyperplane sections of certain Grassmannians. We show that when the simple Lie algebra is not $sl_n, n > 3$, these are the only nontrivial examples, modulo the equivalence of germs of minimal rational curves.

Qingchun Ji Fudan University

Division and Extension Problems for Elliptic Structures

This talk is based on our joint work on extending L^2 theory from complex structures to formally integrable structures. By constructing a resolution of the sheaf of germs of solutions, we study the solvability of Cousin type problems for a class of overdetermined systems originally introduced by L. Hörmander. For the special case of elliptic structures, we will talk about applications to division and extension problems.

Sui-Chung Ng East China Normal University

On the rigidity problems related the real hyperquadrics on complex projective spaces

We are going to introduce a coordinate-free approach to the rigidity problems related to the real hyperquadrics on complex projective spaces. Our first observation is that

the holomorphic maps among such hyperquadrics will preserve some kind of orthogonality on the ambient projective spaces. The restrictions of such maps on various linear subspaces will thus have their images controlled by orthogonality. Together with a hyperplane restriction theorem for local holomorphic maps, we can recover and generalize a number of classical results in this subject and also give a partial confirmation for the gap conjecture regarding the structure of rational proper maps between complex unit balls. The results presented in this talk come from the joint works with Yun Gao.

Feng Rong Shanghai Jiao Tong University

Non-standard holomorphic isometric embeddings and the n -point boundary rigidity

This talk has two independent parts. In the first part, we describe a construction of non-standard holomorphic isometric embeddings of the Poincaré disk into the Siegel upper half-plane, motivated by an example of Mok. In the second part, we present a general n -point boundary rigidity result, generalizing a similar result of Liu and Tang on the unit ball.

Min Ru University of Houston

Recent progress in the Diophantine approximation

In this talk, I will report some progress on the study of Diophantine approximation. In particular, I will discuss the recent joint work with Paul Vojta, as well as the applications of the Ru-Vojta's result by Julie Wang and her collaborators.

Yum-Tong Siu Harvard University

Differential Relations for Multiplier Ideal Sheaves from the Complex Neumann Problem

For sums of squares of real vector fields, Hörmander linked subelliptic estimates to the spanning property of iterated Lie brackets of vector fields. Kohn studied the more complicated analogue of subelliptic $\bar{\partial}$ estimates for weakly pseudoconvex domains, with vector-valued unknowns.

In the weak-solution approach to solving the $\bar{\partial}$ equation, multipliers for the test function are introduced so that estimates hold after multiplication by a multiplier. Kohn used the dual formulation of differential forms instead of vector fields so that

(i) the Lie brackets of vector fields are replaced by differential relations to generate new multipliers and (ii) the spanning property of iterated Lie brackets is replaced by the constant function 1 being a multiplier.

We will focus on recent problems and results concerning Kohn's theory of subelliptic estimates in terms of D'Angelo's condition of finite type. The distant eventual goal is to explore the theory for differential relations to generate multipliers for subelliptic estimates for a general system of equations with compatibility conditions.

Julie Wang Academia Sinica

Vojta's generalized abc Conjecture for algebraic tori and applications over function fields

We first formulate a result concerning Vojta's generalized *abc* conjecture for algebraic tori over function fields with explicitly determinable exceptional sets. As an application, we investigate the Lang-Vojta Conjecture for varieties of log general type that are ramified covers of algebraic tori over function fields. Our methods also apply to the complex situation, allowing us to identify explicit exceptional sets for the corresponding case of Vojta's general *abc* conjecture (complex version) and the Green-Griffith-Lang conjecture. This is a joint work with Ji Guo, Khoa D. Nguyen, and Chia-Liang Sun.

Kwok-Kin Wong HKU

Complex hyperbolicity of subvarieties and Carathéodory metrics

Serge Lang conjectured that a projective manifold X is Kobayashi hyperbolic if and only if all its subvarieties (including X itself) are of general type. Motivated by this, we consider a quasi-projective analogue of the conjecture from the perspective of Carathéodory geometry. In particular, assuming X is covered by a Carathéodory hyperbolic manifold, we will show that the necessity part of Lang's conjecture holds. For the quasi-projective analogue, similar necessity holds if the cover is further assumed to be strongly Carathéodory hyperbolic.

Sai-Kee Yeung Purdue University

Deformation and rigidity of holomorphic the holomorphic tangent bundle of a complex two ball quotient

It is known from the work of Calabi-Vesentini in the 60's that a smooth locally Hermitian symmetric space of non-compact type is locally rigid in the space of deformation of the manifold. A natural question is whether the holomorphic tangent bundle of such spaces can be deformed among holomorphic vector bundles of the same determinant bundle. All the examples of locally Hermitian symmetric spaces in the past for which the question has a known answer have rigid tangent bundle in the above sense, due to a result of Siu in the 80's. In particular, for complex hyperbolic n spaces, Siu showed that there was no deformation for $n > 2$ in the above sense. The case of $n = 2$ is subtle and has been open. We will explore the problem from different perspectives. In the end, we will show that there are examples with non-trivial deformations of the holomorphic tangent bundle in the above sense for some complex two ball quotients.

Xiangyu Zhou Chinese Academy of Sciences

Multiplier submodule sheaves and a criteria of vector bundles being Nakano spositivity

First we'll recall some recent results on multiplier ideal sheaves and converse L^2 theory, then we'll explain some new results on multiplier submodule sheaves associated to singular hermitian metric on holomorphic vector bundles, including a criteria of holomorphic vector bundles being Nakano semipositivity and a solution of Lempert's problem on Nakano semipositivity.