IMR-IoM Workshop on Complex Geometry 2025
12 - 14 February, 2025

Titles and Abstracts

Shan-Tai CHAN Chinese Academy of Sciences

Geometry of local holomorphic maps between Kähler manifolds preserving (p, p)-forms

In this talk, we will explore the geometry of local holomorphic maps between Kähler manifolds that preserve (p, p)-forms, specifically the p-th exterior power of the Kähler forms. We will delve into some rigidity and non-existence theorems for such maps between certain Kähler manifolds. In 1953, E. Calabi established that there does not exist any local holomorphic isometry between complex space forms of different types. Our primary result generalizes Calabi's theorem to local holomorphic maps between finite-dimensional complex space forms that preserve (p, p)-forms, except in the case where the universal cover of the domain is biholomorphic to a complex Euclidean space of complex dimension m, the target is the complex projective space of complex dimension n, and $2 \le p = m < n$. This exceptional case has been addressed by the recent work of Arezzo-Li-Loi (arXiv:2306.16113). This talk is based on my joint work with Yuan Yuan (arXiv:2312.09655).

Kenneth Chung-Tak CHIU HKU

Complex cellular structures, slope-determinant method, and hypersurface coverings of rational points

We will first talk about complex cellular structures introduced by Binyamini and Novikov. We will also mention Bost's slope method and Chen's results on the evaluation map in Arakelov geometry, and mention a bound for the determinant of the evaluation map when rational points are in the same complex cell. We explain how we combine these to estimate the number of hypersurfaces required to cover the regular rational points with bounded Arakelov height on a projective variety.

Cong DING Shenzhen University

Rigidity of smooth Schubert cycles in rational homogeneous spaces of higher Picard number

Given a multiple of Schubert cycle $r[X_w]$ in a rational homogeneous space $X = G/P$, if any subvariety $Z \subset X$ with $[Z] = r[X_w]$ must have $Z = g_1X_w + \cdots + g_rX_w$ for some $g_i(1 \leq i \leq r) \in G$, then $X_w \subset X$ is said to be Schur rigid. In particular, if we only assume that this holds for $r = 1$, then $X_w \subset X$ is said to be homologically rigid. The rigidity of Schubert cycles is a classical problem and there are many known results when X is of Picard number one. In this talk, I will discuss some recent progress of this problem when X is of higher Picard number and X_w is smooth. This is a joint work with Qifeng Li.

Baohua FU Chinese Academy of Sciences

Symplectic singularlties arising from cotangent bundles

I'll report joint works with Jie Liu (AMSS), in which we investigate symplectic singularities arising from the affinization of the cotangent bundle of a smooth variety.

Siqi HE Chinese Academy of Sciences

$Z/2$ harmonic forms, harmonic maps into R-trees, and compactifications of character variety

In this talk, we will explore the connection between the analytic compactification of the moduli space of flat $SL(2, C)$ connections on closed, oriented 3-manifolds defined by Taubes, and the Morgan–Shalen compactification of the $SL(2, C)$ character variety. We will discuss how these two compactifications are related through harmonic maps to R-trees. Additionally, we will discuss several applications of this construction in the analytic aspects of gauge theory. This is joint work with R. Wentworth and B. Zhang.

Zheng HUA HKU

Modular Poisson Structure Associated with Degeneration of Elliptic Curves

Collaborating with Sasha Polishchuk, a canonical Poisson structure has been constructed in a series of papers on the moduli stack of complexes of vector bundles over degenerations of elliptic curves. This is a vast generalization of various classical Poisson structures, such as the Feigin-Odesskii bracket for coherent systems over elliptic curves and the Mukai-Bottacin Poisson structure for vector bundles on Poisson surfaces. During this presentation, I will provide an overview of the diverse applications of this construction in Poisson geometry, integrable systems, and noncommutative geometry.

Wangjian JIAN Chinese Academy of Sciences
Recent progresses of the Analytic MMP

First, we will introduce the background of the Analytic MMP proposed by Song-Tian. Then we will focus on the finite time singularities of the Kähler-Ricci flow, and talk about some recent progresses in this field. Finally, we will talk about some further questions.

Jie LIU Chinese Academy of Sciences
Kawamata—Miyaoka inequality for Q-Fano varieties

The classical Bogomolov—Miyaoka–Yau inequality says that the second Chern class of a canonically polarized projective variety with mild singularities can be bounded from below by its volume. I will present an analogous inequality, called the Kawamata—Miyaoka inequality, for Fano varieties with Picard number one and then I will discuss its applications in the explicit classification of Fano threefolds with Picard number one. This is based on joint works with Haidong Liu and also joint work with Chen Jiang and Haidong Liu.

Jinsong LIU Chinese Academy of Sciences
L^p norm of truncated Riesz transform and an improved dimension-free L^p estimate for maximal Riesz transform

In this talk we will show that the $L^p(\mathbb{R}^d)$ norm of the maximal truncated Riesz transform in terms of the $L^p(\mathbb{R}^d)$ norm of Riesz transform is dimension-free for any $1 < p < \infty$, using maximal estimates for radial multipliers.

Moreover, we show that

$$\|R_j^* f\|_{L^p} \leq \left(\frac{4+\sqrt{2}}{2}\right)^{\frac{2}{p}} \|R_j f\|_{L^p}, \qquad \text{for} \;\; p \geq 2, \quad d \geq 2.$$

As by products of our calculations, we infer the L^p norm contractivity of truncated Riesz transforms R_j^t in terms of R_j, and their accurate L^p norms.

This is a joint work with Petar Melentijević and Jian-Feng Zhu.

Ngaiming MOK HKU

The canonical height for abelian varieties over complex function fields

Let $\mathcal{H}_g$ be the Siegel upper half-plane consisting of g-by-g complex symmetric matrices with positive imaginary parts. For $g \geq 1$ and $n \geq 3$, let $\Gamma_g = \Gamma_g(1) := \mathbb{P}\mathrm{Sp}(g, \mathbb{Z})$ be the Siegel modular group, and $\Gamma_g(n) \subset \Gamma_g$ be the principal congruence subgroup so that $\mathcal{A}_g(n) := \mathcal{H}_g/\Gamma_g(n)$ classifies principally polarized abelian varieties equipped with level-n structures. Write $\pi : \mathfrak{A}_g(n) \to \mathcal{A}_g(n)$ for the associated Kuga family, and $\overline{\pi} : \overline{\mathfrak{A}_g(n)} \to \overline{\mathcal{A}_g(n)}$ for its compactification. Let B be a quasi-projective variety, $f : B \to \mathcal{A}_g(n)$ be a classifying map, which automatically extends to a holomorphic map $F : \overline{B} \to \overline{\mathcal{A}_g(n)}$ for some projective compactification $\overline{B} \supset B$. Pulling back by the classifying map we obtain an abelian scheme $\overline{\pi_f} : F^*\overline{\mathfrak{A}_g(n)} \to \overline{B}$. Write $\mathfrak{A}_f$ for $f^*\mathfrak{A}_g(n)$. Then, $\overline{\pi_f} : \overline{\mathfrak{A}_f} \to \overline{B}$, where $\overline{\mathfrak{A}_f} := F^*\overline{\mathfrak{A}_g(n)}$, is a geometric model of an abelian variety $\mathbf{A}_f$ over the algebraic function field $K := \mathbb{C}(\overline{B})$. Our interest is to estimate the rank of the Mordell-Weil group $\mathbf{A}_f(K)$ of K-rational points in terms of the classifying map $f : B \to \mathcal{A}_g(n)$, under the assumption that $\pi_f : \mathfrak{A}_f \to B$ has no constant parts.

A K-rational point on $\mathbf{A}_f$ equates to a meromorphic section $\sigma : \overline{B} \dashrightarrow \overline{\mathfrak{A}_f}$ of $\overline{\pi_f} : \overline{\mathfrak{A}_f} \to \overline{B}$. In 1991, I introduced the notion of the canonical height on $\mathbf{A}_f(K)$ obtained in case $\dim(B) = 1$ by integrating on $\sigma(B)$ a certain locally homogeneous semipositive d-closed $(1,1)$-form μ on $\mathfrak{A}_g(n)$, first defined for the modular abelian variety $\mathbf{A}_g(n)$ and then in general via the classifying map. $\mathrm{K}er(\mu)$ gives a foliation $\mathscr{F}$ on $\mathfrak{A}_g(n)$ whose local leaves are the images of torsion sections and their topological limits. For a nonsingular compactification $\overline{\pi} : \overline{\mathfrak{A}_g(n)} \to \mathcal{A}_g(n)$ we also proved that μ extends trivially to a closed positive $(1,1)$-current T on $\overline{\mathfrak{A}_g(n)}$. This allowed Mok (1991) and Mok-To (1993) to prove finiteness of the Mordell-Weil group $\mathbf{A}(K)$, $K := \mathbb{C}(\overline{X_\Gamma})$, for a Kuga family $\pi : \mathfrak{A} \to X_\Gamma$ without constant parts over a Shimura variety $X_\Gamma := D/\Gamma$, and to obtain estimates on $\mathrm{rank}(\mathbf{A}_f(K))$, $K := \mathbb{C}(\overline{B})$, when $f : B \to X_\Gamma$ is a generically finite equidimensional holomorphic map, in terms of $\mathrm{Volume}(R_f)$ for the ramification divisor R_f.

There is currently a resurgence of interest on the closed semipositive $(1,1)$-form μ, now commonly called the Betti form, and the underlying Betti map. For $g=1$, Mok-Ng (2023) applied the complex differential-geometric approach to prove a finiteness result on points of Betti multiplicities ≥ 2 of a section $\sigma \in \mathbf{E}(K)$ of infinite order in the case of an elliptic scheme $\pi : \mathfrak{E}_f \to B$ over a quasi-projective curve B, $K := \mathbb{C}(\overline{B})$, a result first obtained by Corvaja-Demeio-Masser-Zannier, which was rendered effective by Ulmer-Urzúa (2021). Our effective result is based on a fundamental first-order real-linear differential equation satisfied by the vertical part η_σ of $d\sigma$ for $\sigma \in \mathbf{E}(K)$, and has the advantage of being applicable in principle to abelian schemes. Corvaja et-al. also proved that in case $g=1$ what we called the "canonical height" in Mok (1991) is actually the Néron-Tate height associated to the standard height obtained from the Fubini-Study metric (via a projective embedding of $\overline{\mathfrak{E}}_f$). The same statement for the general case of $g \geq 1$ is a special case of a result of Gauthier-Vigny (2023) for endomorphisms on polarized schemes of projective varieties. We have now obtained a proof that, starting with an *integral* height defined by a Kähler form Ω on $\mathfrak{A}_f$ admitting a trivial extension as a closed positive $(1,1)$-current T to a nonsingular compactification $\overline{\mathfrak{A}_f}$, the Néron-Tate limiting process gives the "canonical height" as defined by the Betti form provided that a certain condition on Lelong numbers $\nu(T;y)$ for $y \in \overline{\mathfrak{A}_f} - \mathfrak{A}_f$ is satisfied.

Tuen-Wai NG HKU

Brody curves on Fermat surface of degree six

A holomorphic map from the complex line to the n-dimensional complex projective space is called a Brody curve if its spherical derivative is bounded. In 2010, Eremenko applied potential theory to study Brody curves omitting n hyperplanes in general position and showed that these curves have growth order at most one, normal type. In this talk, we will characterize Brody curves on the degree six Fermat surface in the three dimensional complex projective space based on Eremenko's potential theoretical method. This is a joint work with Sai Kee Yeung.

Wenjuan PENG Chinese Academy of Sciences

Parabolic implosion surgery for rational maps

In this talk, I will introduce the parabolic implosion surgery of rational maps and prove the convergence of the sequence of the resulting new rational maps obtained by the surgery. This is a recent joint work with Prof. Guizhen Cui.

Qian TANG HKU

Isomonodromy Equations and the Inverse Monodromy Problem

In this talk, we will introduce our recent work on some isomonodromy equations. We can regard it as a higher-dimensional generalization of Painlevé VI, considering it as an integrable system with monodromy data as first integrals. Our main result is the asymptotic behavior of its generic solution, and we can provide its series solution starting from the monodromy data. We hope to use this method to study the inverse monodromy problem.

Kwok-Kin WONG Shenzhen University

Nonexistence of level structures and volume estimates on Carathéodory manifolds

Compactifying modular curves $\mathbb{H}/\Gamma$ may result in curves of rather different geometric properties. Nevertheless, by taking large coverings, hyperbolicity properties are observed for $\mathbb{H}/\Gamma(N)$ when the level N is sufficiently high. In [Ann. Math. 1989], Nadel studied the above phenomenon on Shimura varieties and showed that certain level structures corresponding to genus $0, 1$ could not exist for Shimura varieties of sufficiently high level. Geometrically this corresponds to the nonexistence of entire holomorphic curve in the interior of the compactification. In other words certain Brody hyperbolicity is observed and it is closely related to the Green-Griffiths-Lang conjecture for the setting. The genus ≥ 2 cases are due to Hwang-To [Math. Ann 2006], where a key ingredient is the volume estimates of subvarieties of Shimura varieties. This estimate has been applied to obtain some recent progresses in functional transcendence theory, especially the resolution of Ax-Schanuel conjecture for Shimura varieties in Mok-Pila-Tsimerman [Ann. Math. 2019].

In this talk, we prove a generalization of the work of Hwang-To to a larger class of manifolds including as new examples the moduli spaces of hyperbolic Riemann surfaces, form the point of view of Carathéodory geometry.

Xiangyu ZHOU Chinese Academy of Sciences

Several Complex Variables at IoM

Will outline a development of several complex variables at Institute of Mathematics, Chinese Academy of Sciences.