Better-behaved GKZ hypergeometric systems and their duality

Lev Borisov

9:30-10:30, Nov 4,5,7

Abstract: I will talk about the version of the Gel'fand-Kapranov-Zelevinsky hypergeometric systems that is best suited for the setting of local toric mirror symmetry. Starting from simple combinatorial data of a lattice polytope Δ, one can define two vector bundles with flat connections over the space of non-degenerate Laurent polynomials with support Δ. Flat sections of these vector bundles are related to the K-theory of toric Deligne-Mumford stacks defined by triangulations of Δ. I will, in particular, discuss recent work, joint with Zengrui Han, which gives an explicit formula for the long-conjectured duality of these systems. I will try to keep the discussion as elementary as possible, with no familiarity with mirror symmetry or GKZ hypergeometric systems assumed.

Poisson geometric approach to exotic cluster structures on simple Lie groups

Misha Gekhtman

10:45-11:45, Nov 4,5,7

Abstract: I will discuss a new approach to building log-canonical coordinate charts for any simply-connected simple Lie group G and arbitrary Poisson-homogeneous bracket on G associated with Belavin-Drinfeld data. Given a pair of representatives r, r' from two arbitrary Belavin–Drinfeld classes, we build a rational map from G with the Poisson structure defined by two appropriately selected representatives from the standard class to G equipped with the Poisson structure defined by the pair r, r'. In the A_n case, we prove that this map is invertible whenever the pair r, r' is drawn from aperiodic Belavin-Drinfeld data and apply this construction to recover the existence of a regular complete cluster structure compatible with the Poisson structure associated with the pair r, r'. A similar construction exists in the case of the dual Poisson Lie groups. The necessary background on Poisson-Lie groups and cluster structures compatible with Poisson brackets. An emergence of generalized cluster structures in the context will also be discussed. (Based on joint work with M. Shapiro, A. Vainshtein and D. Voloshyn.)

Additive categorification for positroid varieties

Matt Pressland

9:30-10:30, Nov 8,11,12

Abstract: The totally nonnegative Grassmannian is an important object in several stories, including Lusztig's total positivity, and the calculation of scattering amplitudes via the amplituhedron. It has a cell decomposition, described by Postnikov, in which each cell is obtained by intersecting the totally nonnegative Grassmannian with a particular subvariety of the full Grassmannian: a so-called open positroid variety. A useful tool in studying totally positive spaces is Fomin–Zelevinsky's theory of cluster algebras, and a recent result of Galashin and Lam is that the coordinate ring of (the cone on) an open positroid variety indeed has a natural cluster algebra structure. In this lecture series, I will describe this cluster structure and explain how to use representation-theoretic techniques to understand it, setting up a dictionary between the combinatorics and the algebra. Some of the results here are joint with anak and King. Galashin and Lam actually produce two (isomorphic, but usually not equal) cluster algebra structures on each positroid variety, which correspond in algebraic terms to the choice between left and right modules. An application of the categorification is to prove a precise relationship, called quasi-coincidence, between these two cluster algebra structures, originally conjectured by Muller and Speyer in 2017.

Quivers with relations for symmetrizable Cartan matrices

Christof Geiss

10:45-11:45, Nov 8,11,12

Abstract: This series of talks is mainly based on a series of papers which I published between 2016 and 2020 together with Bernard Leclerc and Jan Schrer. Let C be a (generalized) symmetrizable Cartan matrix with a symmetrizer D and an orientation Ω. If C is symmetric, we may assume that D is trivial and we can associate to this data over any field K a path algebra and the corresponding preprojective algebra. However, if C is not symmetric, similar constructions by Dlab and Ringel were only possible over certain fields which are not algebraically closed. In a series of papers we explore the possibility to replace field extensions by truncated polynomial rings, or by rings of formal power series.

Lecture 1: We introduce for any field K for the data (C, D, Ω) a finite dimensional 1-Iwanaga Gorenstein algebra $H_F(C, D, \Omega)$ which is our replacement for a path algebra, and a generalized preprojective algebra $\Pi_F(C, D, \Omega)$. The latter one is independent of the choice of the orientation and is finite dimensional if and only if C is of finite type. Both algebras are defined explicitly in terms of a quiver with relation, and we explore their basic homological properties, locally free modules and dependance on D.

Lecture 2: We review to what extent a construction of Schofield can be extended, over an algebraically closed field K to our algebras $H_F(C, D, \Omega)$ in order to construct the positive part of the corresponding Kac-Moody Lie algebra of type C. This allows us to get for all acyclic cluster algebras of finite type a geometric interpretation of the cluster expansion with respect to an acyclic seed.

Lecture 3: We study, again over an algebraically closed field F, for every rank vector the irreducible components of top dimension of the representation varieties of locally free, E-filtered representations. Following ideas of Kashiwara and Saito, the union of all those sets of irreducible components (of top dimension) have naturally the structure of the crystal graph $B(-\infty)$ associated to C. One can also construct a convolution algebra a la Lusztig, but there is only a *surjective* homomorphism onto the enveloping algebra $U(\mathfrak{n}_+(C))$.

Cluster algebras and symplectic geometry

Gao honghao

14:30-15:30, Nov 12,14,15

Abstract: Cluster algebra is a very rich structure that also appears in many other subjects. This lecture series is devoted to the connection between cluster algebras and symplectic geometry. Legendrian knots and their Lagrangian fillings are important geometric objects in 4 dimensional symplectic geometry. The aim of the lecture series is to explain how the cluster algebra emerges when studying invariants for Legendrian knots and how it can be used to classify exact Lagrangian fillings by surveying works in literature.

Introduction to total positivity

Bao Huanchen

9:30-10:30, Nov 13,14,15

Abstract: An invertible $n \times n$ real matrix is called totally positive if all its minors are positive. The study of such matrices dates back to Schoenberg and Grantmacher-Krein in 1930s. The theory has been generalized by Lusztig in 1994 to arbitrary split reductive connected groups (from general linear groups). This generalization depends on deep results from the theory of canonical bases arising from quantum groups. Since then, the theory of total positivity has found numerous applications, including cluster algebras, higher Teichmuller theory, the physics of scattering amplitudes, etc. The goal of this course is give an introduction to the theory of total positivity.

Introduction to additive categorification of cluster algebras with coefficients

Bernhard Keller

10:45-11:45, Nov 13,14,15

Abstract: Recall that a cluster algebra is a commutative algebra endowed with a rich combinatorial structure including a set of distinguished generators, the cluster variables, which are grouped into overlapping subsets of fixed (usually finite) cardinality, the clusters. In additive categorification, one tries to link the combinatorics of the cluster variables in a given cluster algebra to the combinatorics of the rigid indecomposable objects in a suitable triangulated category endowed with a 2-Calabi-Yau structure. In this lecture series, we will start by explaining this link in the case of cluster algebras associated with finite (simply laced) root systems or, equivalently, with Dynkin quivers. Here, we will only need the notion of a quiver representation and basic facts from Lie theory and algebraic geometry. In order to lift more of the combinatorics to the categorical level, we will then introduce the cluster category of more general acyclic quivers. In the final part of the series, we will consider cluster algebras with coefficients arising in geometric contexts, for example in Fock-Goncharov's approach to higher Teichmuller theory. We will construct additive categorifications of such cluster algebras using Yilin Wu's Higgs categories, which are no longer triangulated but still extriangulated in the sense of Nakaoka-Palu. Time permitting, we will also present an alternative approach due to Merlin Christ.

All the talks are in ***Room 210 of Run Run Shaw Building.***