Abstracts

Olga Balkanova, Steklov Mathematical Institute

Title: *The second moment of symmetric square L-functions over Gaussian integers*

In this talk, we describe the main ideas behind the proof of the upper bound for the second moment of Maass form symmetric square L-functions defined over Gaussian integers. Furthermore, we show how our estimate can be used in the prime geodesic theorem.

Gebhard Böckle, University Heidelberg

Title: *Deformation rings and images of Galois representations*

Let G be a connected reductive almost simple group over the Witt ring $W(F)$ of a finite field F of characteristic p, and let R and R' be complete noetherian local $W(F)$-algebras with residue field F. Under a mild condition on the prime p in relation to structural constants of G, in joint work with $S.$ Arias de Reyna we show the following results:

(1) Every closed subgroup of $G(R)$ with full residual image $G(F)$ is a conjugate of a group $G(A)$ for A a closed subring of R that is local and has residue field F.

(2) Every continuous surjective homomorphism $G(R) \to G(R')$ is, up to conjugation, linear, i.e., induced from a ring homomorphism $R \to R'$.

In particular the reduction map $G(R) \to G(F)$ has R as its universal deformation ring.

In the talk I plan to present the axiomatic framework that is at the basis of these results, explain how (1) (conjecturally) applies to natural compatible families of Galois representations for most primes p, and indicate some proofs in the simplest case SL_n, where the result on universal deformations was known by work of Dorobisz and Eardley-Manoharmayum.

Anna Cadoret, Sorbonne Université

Title: *Etale ultraproduct coefficients and applications*

The aim of this talk is to give an overview of the theory of (constructible) etale sheaves with ultraproduct coefficients. This theory - built out from the one of etale sheaves with $\overline{\mathbb{F}}_\ell$-coefficients - parallels and complements the theory of etale sheaves with $\overline{\mathbb{Q}}_\ell$-coefficients. Combining both type of coefficients, one gets transparent proofs for most of the asymptotic phenomena for $\overline{\mathbb{Z}}_\ell$-models in compatible families (of $\overline{\mathbb{Q}}_\ell$-local systems or pervese sheaves) predicted by motivic conjectures. This is a joint work with Weizhe Zheng.

Peter Graef, University Heidelberg

Title: *Computing L-invariants via the Greenberg-Stevens formula*

Going back to the exceptional zero conjecture of Mazur, Tate and Teitelbaum, p-adic L-invariants attached to classical newforms have proven to be of great arithmetic interest. These L-invariants are in fact purely local objects, in the sense that they only depend on the p-adic local Galois representation attached to the newform under consideration.

In this talk, I will discuss joint work with S. Anni, G. Böckle and Á. Troya concerning the explicit computation of (valuations of) p-adic L-invariants. This is based on a famous formula due to Greenberg-Stevens and work of Lauder and Vonk on computing the reverse characteristic series of the U_p-operator on overconvergent modular forms. After explaining our method, I will give computational evidence for relations between valuations of L-invariants of different levels and weights for small primes p. Finally, I will give an outlook on work in progress with A. Conti towards explaining these relations.

Zilong He, Southern University of Science and Technology

Title: *On 2-universal integral quadratic forms over local fields*

Let F be an algebraic number field. Recently, Xu and Zhang give the necessary and sufficient conditions for a binary (resp. ternary) indefinite quadratic $\mathcal{O}_F$-lattice to be locally universal, but not globally. Motivated by their work, we give an analogy of Conway and Schneeberger's 15-theorem for 2-universal quadratic lattices over local fields and the equivalent conditions for a quaternary locally 2-universal quadratic $\mathcal{O}_F$-lattice not to be globally 2-universal.

In this talk, we will review known results on universal quadratic forms and then present our results. Also, we will briefly introduce the representation theory of quadratic forms over dyadic local fields, developed by Beli. This is a joint work with Hu Yong and Xu Fei.

Yueke Hu, Tsinghua University

Title: *Refined Petersson/Kuznetsov trace formulae and their applications*

In this talk I will introduce refined Petersson/Kuznetsov trace formulae in the level aspect, whose spectral side picks out only newforms, and the spectral sum is shorter compared to the classical formulae. Then I will talk about their applications to the first moment of the Rankin-Selberg L-functions, and the relevant subconvexity bound.

Bingrong Huang, Shandong University

Title: *Quantum variance for automorphic forms*

In this talk, we will present results on the quantum variances for Eisenstein series and for dihedral Maass forms on modular surfaces. The resulting quadratic forms are compared with the classical variance (Ratner) and the quantum variance for cusp forms (Luo–Sarnak). They coincide after inserting certain subtle arithmetic factors, including the central values of certain L-functions. (Based on joint work with Stephen Lester.)

Bo-Hae Im, Korea Advanced Institute of Science and Technology

Title: *Rational curves on the quotient varieties of abelian varieties and its application to the rank problem of abelian varieties*

In this talk, we discuss a criterion on the existence of rational curves on the quotient varieties of abelian varieties by some finite group of automorphisms and its application on the growth of the rank of abelian varieties. Also we talk about some non-existence of rational curves on the Kummer varieties of certain abelian varieties. As a consequence, Pirola's theorem about rational curves on sufficiently general Kummer varieties of dimension ≥ 3 generalizes to Kummer varieties parametrized by certain Shimura varieties. The main result of non-existence is a joint work with M. Larsen and S. Zhan.

Jangwon Ju, University of Ulsan

Title: *Ternary quadratic forms representing the same integers*

In 1997, Kaplansky conjectured that if two positive definite ternary quadratic forms with integer coefficients have perfectly identical integral representations, then they are isometric, both regular, or included either of two families of ternary quadratic forms. In this talk, we prove the existence of pairs of ternary quadratic forms representing the same integers which are not contained in Kaplansky's list.

Daejun Kim, The University of Hong Kong

Title: *The pentagonal theorem of sixty-three and generalizations of Cauchy's lemma*

The study of writing (or representing) integers as sums of polygonal numbers has a long history, which dates back as far as Fermat, who conjectured in 1638 the famous Polygonal Number Theorem. In this talk, we briefly review the history of this study, and we focus on representability of integers as sums of pentagonal numbers. In particular, the pentagonal theorem of 63 will be described, which states that a sum of pentagonal numbers represents every non-negative integer if and only if it represents 1, 2, 3, 4, 6, 7, 8, 9, 11, 13, 14, 17, 18, 19, 23, 28, 31, 33, 34, 39, 42, and 63. We also introduce a method to obtain a generalized version of Cauchy's lemma using representation of binary integral quadratic forms by quaternary quadratic forms, which plays a crucial role in proving the results. This is a joint work with Jangwon Ju.

Mingyu Kim, Sungkyunkwan University

Title: *Tight universal sums of polygonal numbers*

For $m \geq 3$, let $p_m(x) = \dfrac{(m-2)x^2 - (m-4)x}{2}$. A polynomial of the form

$$p_m(\mathbf{a}) = p_m(\mathbf{a})(x_1, x_2, \ldots, x_k) = a_1 p_m(x_1) + a_2 p_m(x_2) + \cdots + a_k p_m(x_k),$$

where $\mathbf{a} = (a_1, a_2, \ldots, a_k) \in \mathbb{N}^k$ is called a k-ary m-gonal form. The smallest positive integer represented by $p_m(\mathbf{a})$ is called the minimum of $p_m(\mathbf{a})$. In this talk, we study the classifications of m-gonal forms representing every positive integer greater than its minimum.

Alex Kontorovich, Rutgers University

Title: *Length Saturation for Zariski Dense Groups*

We study the length set for Zariski dense, punctured covers of the modular surface, proving an asymptotic local-global phenomenon. This is joint work with Xin Zhang.

Jialun Li, University of Zurich

Title: *Counting and equidistribution of compact maximal flats*

I will talk about the counting and equidistribution of compact maximal flats on locally symmetric spaces $SL(d,\mathbb{Z})\backslash SL(d,\mathbb{R})/SO(d)$. As an application, this implies a counting result of orders in totally real number fields of degree d. These results generalise counting closed geodesics in the modular curve and counting orders in real quadratic fields. The equidistribution is proved by using an idea of Roblin, an orbital counting result of Gorodnik-Nevo and a weak version of non-escape of mass for diagonal flow. The talk is based on joint work with Thi Dang.

Haohao Liu, École normale supérieure (Paris)

Title: *Siegel's theorem via the Lawrence-Venkatesh method*

In 1983, Faltings proved the Mordell's conjecture that a curve of genus greater than 1 over a number field has only finitely many rational points by Parshin's trick. In 2018, Lawrence and Venkatesh gave a new proof by associating a Galois representation to each rational point. The variation of the Galois representations is encoded by a local period map, induced by the Gauss-Manin connection. If one can show that this variation is "big" by comparing it with topological monodromy, then the finiteness of rational points follows. Thus, the problem is now reduced to constructing a particular family of varieties of full monodromy. We will illustrate their method with Siegel's theorem, which is simpler than the case of Mordell's conjecture.

Jingbo Liu, Texas A&M University-San Antonio

Title: *On a Waring's problem for Hermitian lattices*

Assume E is an imaginary quadratic field and $\mathcal{O}$ is its ring of integers. For each positive integer m, let I_m be the free Hermitian lattice of rank m over $\mathcal{O}$ having an orthonormal basis. For each positive integer n, let $\mathfrak{S}_{\mathcal{O}}(n)$ be the set of all Hermitian lattices of rank n over $\mathcal{O}$ that can be represented by some I_m. Denote by $g_{\mathcal{O}}(n)$ the smallest positive integer g such that each Hermitian lattice in $\mathfrak{S}_{\mathcal{O}}(n)$ can be represented by I_g. In this talk, we shall provide an explicit upper bound for $g_{\mathcal{O}}(n)$ for all imaginary quadratic fields E and positive integers n.

Chung Pang Mok, SooChow University

Title: *On certain special values of L-functions associated to elliptic curves and real quadratic fields*

The Birch and Swinnerton-Dyer conjecture predicts that, the special values of L-functions associated to elliptic curves, suitably normalized, is the square of a rational number. For certain class of special values associated to elliptic curves and real quadratic fields, this could be proved using the Gross-Zagier formula, in the generalized form due to Shouwu Zhang. Finally, this result is used to improve upon a p-adic Gross-Zagier type formula, due originally to Bertolini-Darmon. The results of this talk appeared in the paper: Transactions of the AMS, volume 374 (2021), 1391-1419.

Byeong-Kweon Oh, Seoul National University

Title: *Recoverable lattices*

A (positive definite and integral) lattice L is called recoverable if any lattice representing all proper sublattices of L represents L itself. If there is a lattice M that represents all sublattices of L except for L itself, then L is called irrecoverable, and M is called an isolation of L (from its sublattices). In this talk, we find some (ir)recoverable lattices and various isolations of irrecoverable lattices.

Zhi Qi, Zhejiang University

Title: *Moments of Central L-values for Maass Forms over Imaginary Quadratic Fields*

I will talk about the twisted moments of central L-values for $GL(2)$ Maass forms over imaginary quadratic fields. As a direct consequence, it is shown that at least 33% of such central L-values do not vanish. This is joint work with Sheng-chi Liu.

Nicolas de Saxcé, Université Paris-Nord

Title: *Arithmetic groups and diophantine approximation*

We shall explain how to use the theory of arithmetic groups to develop a theory of diophantine approximation on generalized flag varieties, varieties that can be obtained as a quotient of a semisimple algebraic Q-group by a parabolic Q-subgroup. This will allow us to derive some generalizations of celebrated theorems of Khintchine, Thue-Siegel-Roth-Schmidt and Kleinbock-Margulis in this more general context. An emphasis will be made on the simplest case of grassmannian varieties, where one studies approximations of subspaces of R^d by rational subspaces.

Kush Singhal, The University of Hong Kong

Title: *Near-miss Identities and Computing Spinor Genera of Ternary Quadratic Forms with Congruence Conditions*

It is conjectured that the theta series corresponding to spinor class number 1 lattice cosets satisfy certain interesting identities involving unary theta functions, which allows us to give explicit formulae for the number of representations by certain quadratic forms with congruence conditions. In this short talk, I will discuss briefly the problem of finding such identities, as well as finding spinor class number 1 ternary quadratic lattice cosets. I will then touch upon the problem of computing the genus and spinor genus of ternary quadratic lattice cosets in general.

Jincheng Tang, The University of Hong Kong

Title: *An introduction to the sum-product phenomena and some of their applications*

We will talk about the ubiquitous sum-product phenomena, i.e. the expansion of $A + A$ and $A.A$ (under some measure) for a subset A of a ring R. For some rings such as $\mathbb{Z}/n\mathbb{Z}$ the sum-product phenomenon was well studied and formulated as the sum-product theorem, while it remains an open problem for some other rings. We will introduce the history of research in sum-product phenomena, as well as some useful techniques in the proof of the theorems and some of their applications (e.g. in exponential sum estimates).

Ryoko Tomiyasu, Kyushu University

Title: *Packing theory derived from phyllotaxis and products of linear forms*

Since the phyllotaxis study by the Bravais brothers, it has been known that the golden angle method can be used to generate dense packings on surfaces with circular symmetry. Recent work by the speaker has made it possible to apply the method to more general surfaces and higher dimensional manifolds. The discrete part of the problem can be solved as a problem of geometry of numbers including the Markoff theory. The resulting method can be used for pattern formation and growth models in nature (This is a joint work with Mr. S. E. Graiff Zurita and Prof. B. Kane).