Abstracts

Sara Arias-de-Reyna, Universidad de Sevilla

Title: *Galois families of weight one modular forms and field arithmetic*

Modular forms are holomorphic functions on the upper half-plane which display some symmetry with respect to the action of a subgroup of $SL(2, \mathbb{Z})$. Surprisingly, it turns out that some modular forms encode a great deal of arithmetic information about certain field extensions of the rational numbers. This connection between arithmetic and modular forms has been fruitfully exploited to solve problems in number theory.

In this talk we will make use of the interplay between these two subjects to provide an application of field arithmetic to the existence of certain families of weight one modular forms.

This is joint work with Francois Legrand and Gabor Wiese.

Lior Bary-Soroker, Tel Aviv University

Title: *Probabilistic Galois Theory, the square discriminant case*

Consider the easiest model of random polynomials with integers coefficients, where we fix the degree $n = \deg f$ and we choose the coefficients uniformly at random from a large box and let its size L go to infinity. Probabilistic Galois theory, in its naviest form, asks for the distribution of the Galois group of f.

In 1936, Van der Waerden proved that the Galois group of the polynomial is the full symmetric group S_n asymptotically almost surely. He conjectured that the second most probable group is S_{n-1}. This conjecture has seen a lot of progress along the years. Recently, there was a big breakthrough by Bhargava who showed that the second most probable group is either S_{n-1} or A_n. The breakthrough main achievement was the new upper bound C/L on the probability of the Galois group being A_n. One may believe that, in fact, the probability for A_n must be much smaller; the goal of the talk is to discuss what should be the probability for A_n, to give lower bounds, and new heuristics.

Jan Hendrik Bruinier, Technische Universitat Darmstadt

Title: *Arithmetic volumes of unitary Shimura varieties*

The geometric volume of a unitary Shimura variety can be defined as the self-intersection number of the Hodge line bundle on it. It represents an important invariant, which can be explicitly computed in terms of special values of Dirichlet L-functions. Analogously, the arithmetic volume is defined as the arithmetic self-intersection number of the Hodge bundle, equipped with the Petersson metric, on an integral model of the unitary Shimura variety. We show that such arithmetic volumes can be expressed in terms on logarithmic derivatives of Dirichlet L-functions. This is joint work with Ben Howard.

Yonggao Chen, Nanjing Normal University

Title: *On a conjecture of Erdős and Lewin*

A set A of positive integers is called d-complete if every sufficiently large integer is the sum of distinct terms taken from A such that no one divides the other. In this talk, we answer two questions of Erdős and Lewin partially and settle a conjecture of Erdős and Lewin on d-complete sequences affirmatively. We also pose two conjectures for further research. This is a joint work with Wang-Xing Yu.

Kenneth Chung Tak Chiu, University of Toronto

Title: *Arithmetic sparsity in mixed Hodge settings*

We obtain a subpolynomial count of integral Laurent polynomials with fixed data. We also obtain a more general subpolynomial counting result in a setting involving mixed period mapping. It uses simplicial methods in singularity theory and is based on the recent theory developed by Brunebarbe-Maculan and Ellenberg-Lawrence-Venkatesh (in the pure Hodge setting) about constructing a certain cover and counting points in it.

Daniel El-Baz, Technische Universität Graz

Title: *Primitive rational points on expanding horospheres: effective joint equidistribution*

I will report on ongoing work with Min Lee and Andreas Strömbergsson. Using techniques from analytic number theory, spectral theory, geometry of numbers as well as a healthy dose of linear algebra and building on a previous work by Bingrong Huang, Min Lee and myself, we furnish a new proof of a 2016 theorem by Einsiedler, Mozes, Shah and Shapira. That theorem concerns the equidistribution of primitive rational points on certain manifolds and our proof has the added benefit of yielding a rate of convergence. It turns out to have several (perhaps surprising) applications to number theory and combinatorics, which I shall also discuss.

Ya-qing Hu, Morningside Center of Mathematics, Chinese Academy of Sciences

Title: *Reflecting numbers of various types*

A nonzero integer n is called a reflecting number of type (k, m) if

$$n - t^m = u^k,\ n + t^m = v^k$$

have a rational solution $(t, u, v) \in \mathbb{Q}^* \times \mathbb{Q} \times \mathbb{Q}$. In particular, reflecting numbers of type (2, 2) are all congruent numbers and thus will be called reflecting congruent numbers. We can show that all prime numbers $p \equiv 5 \bmod 8$ are reflecting congruent and in general for any integer $k \geq 0$ there are infinitely many square-free reflecting congruent numbers in the residue class of 5 modulo 8 with exactly $k+1$ prime divisors. Moreover, we conjecture that all prime congruent numbers $p \equiv 1 \bmod 8$ are reflecting congruent. In addition, we show that there are no reflecting numbers of type (k, m) if $\gcd(k, m) \geq 3$.

Junxian Li, Universität Bonn

Title: *Hardy-Littlewood problems with almost primes*

The Hardy-Littlewood problem asks for the number of representations of an integer as the sum of a prime and two squares. We consider the Hardy-Littlewood problem where the two squares are restricted to squares of almost primes. A lower bound of the expected order of magnitude can be obtained. The same technique also shows that there are infinitely many primes that can be written as sum of two almost prime squares plus one. We also discuss the problem of writing an integer as the sum of a smooth number and two almost prime squares.

Siu Hang Man, University of Cologne

Title: *A density theorem for Sp(4)*

We prove a density theorem that bounds the number of automorphic forms of level q for the group Sp(4) that violates the Ramanujan conjecture relative to the amount by which they violate the conjecture, which goes beyond Sarnak's density hypothesis. The proof relies on a relative trace formula of Kuznetsov type, and non-trivial bounds for certain Sp(4) Kloosterman sums.

Hasan Saad, University of Virginia

Title: *Distribution of values of Gaussian hypergeometric functions*

In the 1980's, Greene defined *hypergeometric functions over finite fields* using Jacobi sums. The framework of his theory establishes that these functions possess many properties that are analogous to those of the classical hypergeometric series studied by Gauss and Kummer. These functions have played important roles in the study of Apéry-style supercongruences, the Eichler-Selberg trace formula, Galois representations, and zeta-functions of arithmetic varieties. We study the value distribution (over large finite fields) of natural families of these functions. For the ${}_2F_1$ functions, the limiting distribution is semicircular (i.e. $SU(2)$), whereas the distribution for the ${}_3F_2$ functions is the *Batman* distribution for the traces of the real orthogonal group O_3. Furthermore, we find an explicit bound on the error in the distribution of the ${}_3F_2$ using Rankin-Cohen brackets in harmonic Maass forms.

Boya Wen, University of Wisconsin - Madison

Title: *A Gross-Zagier Formula for CM cycles over Shimura Curves*

This talk is based on my thesis work and joint work (in progress) with Congling Qiu. I will introduce a Gross-Zagier formula for CM cycles over Shimura curves. The formula connects the global height pairing of CM cycles in Kuga varieties over Shimura curves with the derivatives of the L-functions associated to weight-$2k$ modular forms. As a key original ingredient of the proof, I will introduce some harmonic analysis on local systems over graphs, including an explicit construction of Green's function, which we apply to compute suitable integral extensions of the CM cycles, and local intersections at primes where the quaternion algebra associated with the Shimura curve ramifies.

Gabor Wiese, Université du Luxembourg

Title: *Splitting fields of* $X^n - X - 1$ *and modular forms*

In his article 'On a theorem of Jordan', Serre considered the family of polynomials $f_n(X) = X^n - X - 1$ and the counting function of the number of roots of f_n over the finite field F_p, seen as function in p. He explicitly showed the 'modularity' of this function for $n = 3, 4$.

In this talk, I report on joint work with Alfio Fabio La Rosa and Chandrashekhar Khare, in which we treat the case $n = 5$ in several different ways.

Ping Xi, Xi'an Jiaotong University

Title: *Lang–Trotter conjecture for CM elliptic curves*

The Lang–Trotter conjecture predicts a striking asymptotic formula on the number of primes $p \leq x$ such that the trace of Frobenius of an elliptic curve at p is fixed by a given integer. Quite recently, we are able to obtain an upper bound for the counting function, in the CM case, which reaches the correct order of magnitude (as conjectured) for the first time. Moreover, we also show how to use the Hardy–Littlewood conjecture on prime in quadratic progressions to give a conditional proof of the CM Lang–Trotter conjecture with fully explicit constants. The tools will include sieve methods, laws of reciprocity, exponential sums, as well as some other classical arguments in analytic number theory. This is a joint work with Daqing Wan (UCI).

Bin Xu, Tsinghua University

Title: *Arthur's conjectures for symplectic and orthogonal similitude groups*

Arthur (1989) conjectured that the discrete spectrum of automorphic representations of a connected reductive group over a number field can be decomposed into A-packets, in terms of which he also conjectured a multiplicity formula. In this talk I will give an introduction to these conjectures and report on the progress for symplectic and orthogonal similitude groups based on the works of Arthur and Moeglin for classical groups.

Lilu Zhao, Shandong University

Title: *Translation invariant quadratic forms and dense sets of primes*

In this talk, we shall briefly introduce the recent development in additive combinatorics on quadratic equations with variables restricted in dense subsets of integers. We mainly focus on a class of translation invariant quadratic forms f in ten variables. In particular, we prove that f has non-trivial zeros with variables restricted in a dense subsets of primes.