HKU Number Theory Days 2025

Titles and Abstracts

Speaker: Kam Cheong Au (University of Cologne)

Title: $\mathbb{Q}$-relations between colored multiple zeta values

Abstract: Multiple zeta values (MZV), defined by

$$\zeta(s_1, \cdots, s_k) = \sum_{n_1 > n_2 > \cdots > n_k \geq 1} \frac{1}{n_1^{s_1} \cdots n_k^{s_k}}$$

are generalizations of Riemann zeta function. They possess a rich algebraic structure, such as shuffle product and stuffle product, leading to many linear relations between them. These two products are also predicted to exhaust all linear relations. On the other hand, linear relations between colored MZV, which are MZVs twisted by N-th roots of unity, have been difficult to find exhaustively: shuffle and stuffle no longer provides the complete set of relations.

In this talk, I give an interpretation of colored MZV using polylogarithm at algebraic numbers differing from N-th root of unity. This allowing us to find more relations than the obvious shuffle and stuffle relations. Moreover, for $N = 4, 6, 8$, this method also seems to provide all relations.

Speaker: Henrik Bachmann (Nagoya University)

Title: sl2-algebras and multiple Eisenstein series

Abstract: An sl2-algebra is an algebra A together with an injective Lie algebra homomorphism from the 3-dimensional Lie algebra sl2 into its Lie algebra of derivations Der(A). A classical example of such an algebra is the algebra of quasimodular forms, which is equipped with three prominent derivations satisfying the same commutator relations as the generators of sl2. Quasimodular forms, on the other hand, also have a, partly not fully understood, connection to the theory of multiple zeta values (MZVs). This connection can be explained by a hybrid of MZVs and classical Eisenstein series, called multiple Eisenstein series. The algebra of multiple Eisenstein series contains the algebra of quasimodular forms as a subalgebra. In this talk, we present a conjectural picture suggesting that the sl2-algebra structure of quasimodular forms has a natural extension to the algebra of multiple Eisenstein series. While doing this, we will also give a gentle introduction to the theory of multiple zeta values.

Speaker: Olga Balkanova (Steklov Mathematical Institute)

Title: A Diophantine divisor problem

Abstract: We discuss a new type of restricted divisor problem that is closely related to both the classical Dirichlet divisor problem and the Hardy-Littlewood problem of counting lattice points in a right triangle. The latter problem depends crucially on the arithmetic (Diophantine) properties of the slope of the triangle. A similar phenomenon holds for our divisor problem, which we analyze using analytic properties of certain Dirichlet series that are expressed in terms of infinite sums of Hecke zeta functions with Grossencharacters weighted by the Gauss hypergeometric function. This is joint work with W. Duke and D. Frolenkov.

Speaker: Koustav Banerjee (University of Cologne)

Title: Hyperbolicity of Jensen polynomials and a family of inequalities for sequences

Abstract: In this talk, I will discuss the state of the art regarding hyperbolicity of Jensen polynomials related to partition statistics through the lens of Griffin, Ono, Rolen, and Zagier's grand . Subsequently, I will present a unified framework to prove (asymptotically) certain combinatorial inequalities, for example, log-concavity, higher Turán among many others, for a class of sequences.

This is an ongoing joint work with Kathrin Bringmann and Larry Rolen.

Speaker: Siegfried Böcherer (University of Mannheim)

Title: On denominators of some L-values, when twisted by characters

Abstract: Critical values of L-functions attached to modular forms have been investigated from several points of view, in particular algebraicity and p-adic interpolation. We are interested in the denominators of critical values after twisting by characters. Such denominators should be (after suiatble normalization) related to congruence primes. This is known as long as we consider only denominators "away" from the conductor of the Dirichlet character in question. We will discuss the conductor part of the denominators. We rely on the doubling method as a source of integrality.

Speaker: Wai Kiu Chan (Wesleyan University)

Title: Can we recover an integral quadratic forms by representing all its proper subforms?

Abstract: An integral quadratic form Q is called recoverable if any integral quadratic forms representing all the proper subforms of Q must represent Q. Which quadratic forms are recoverable? For indefinite forms, we have a very satisfactory answer: all indefinite quadratic forms are recoverable. However, not all positive definite quadratic forms are recoverable, and there are examples of infinite families of positive definite irrecoverable quadratic forms. In this talk, I will discuss these results which are based on recent joint works with Byeong-Kweon Oh and with Felipe Valdes Gonzales.

Speaker: Stephen Kwok Kwong Choi (Simon Fraser University)

Title: Limiting Behavior of Rudin-Shapiro Sequence Autocorrelations

Abstract: In this talk, we study the ℓ^1, ℓ^2, and ℓ^∞ moments of $C_m(k)$ which is the aperiodic autocorrelations of Rudin-Shapiro sequences and discuss the results that $\sum_{0<k\leq x}(C_m(k))^2$ and $\max_{0<k\leq x}|C_m(k)|$ are of the same order as x^2 and $x^{\log_2(|\lambda|)}$, respectively, where λ is the real root of x^3-x^2-2x+4. We also give asymptotic upper and lower bounds for $\sum_{0<k\leq x}|C_m(k)|$. These findings generalize the results of Littlewood, Choi, Katz, van der Linden, and Allouche et al. Analogous results for periodic autocorrelation are also obtained.

Furthermore, we construct and study a continuous analogue of $C_m(k)$ with domain $[0,1]$ and show that the maximum value of this function occurs uniquely at $x=2/3$, which supports our conjecture that the maximum value of $|C_m(k)|$ occurs uniquely at some k_m^* with $\lim_{m\to\infty}\frac{k_m^*}{2^m}=\frac{2}{3}$.

This work is a collaboration with Daniel Tarnu.

Speaker: Gergely Harcos (Alfred Renyi Institute of Mathematic)

Title: A new zero-free region for Rankin–Selberg L-functions

Abstract: I will present a new zero-free region for all GL(1)-twists of GL(m)×GL(n) Rankin–Selberg L-functions. The proof is inspired by Siegel's celebrated lower bound for Dirichlet L-functions at s=1. Time permitting, I will also discuss some applications and recent developments. Joint work with Jesse Thorner.

Speaker: Yujiao Jiang (Shandong University)

Title: On Hypothesis H of Rudnick and Sarnak

Abstract: The generalized Ramanujan conjecture (GRC) is one of the foundational problems in modern number theory. Hypothesis H can often substitute for the GRC in some analytic applications. In this talk, we will discuss our recent progress on this hypothesis, along with the applications of this work to other problems in analytic number theory.

Speaker: Daejun Kim (Korea University)

Title: Sums of generalized polygonal numbers of almost prime length

Abstract: Extending the Lagrange's four-square theorem, it is expected that every sufficiently large integer congruent to 4 modulo 24 can be written as a sum of four squares of prime numbers. It has been shown that such integers can be expressed as a sum of four squares of integers, each with fewer than five prime factors. In this talk, we discuss an analogous problem concerning sums of three generalized m-gonal numbers, where parameters are restricted to integers with a bounded number of prime divisors. With some restriction modulo 30, we show that a density one set of integers can be represented as such a sum, where the parameters are restricted to have at most 6361 prime factors. This is joint work with Soumyarup Banerjee and Ben Kane.

Speaker: Eren Mehmet Kıral (Keio University/ RIKEN)

Title: Kloosterman Sums for SL_3 long word element

Abstract: Using the reduced word decomposition of the long word element of the Weyl group element of SL_3, we give a nice expression for the long word Kloosterman sum. This formula also allows us to write the triple divisor function $\sigma_{u,v,w}(n) = \sum_{n=abc} a^u b^v c^w$ as a Double Dirichlet series of finite exponential sums. Classical Kloosterman sums, along with their matrix formulation will be introduced. This is joint work with Maki Nakasuji of Sophia University (Tokyo).

Speaker: Krishnarjun Krishnamoorthy (Beijing Institute of Mathematical Sciences and Applications)

Title: On Moments of non-normal number fields

Abstract: Let K be a number field and let $a_K(m)$ be the number of integral ideals of norm equal to m in K. We provide asymptotic evaluations for the moments of $a_K(m)$. This is partly joint work with Kalyan Chakraborty.

Speaker: Siu Hang Man (Univerzita Karlova)

Title: Explicit Kuznetsov trace formula for higher rank groups

Abstract: The Kuznetsov trace formula is an indispensable tool in studying Maaß forms on $GL(2)$. While abstract trace formulae exist very generally for many groups, it takes a significant amount of work to make them explicit enough for the applications in analytic number theory. In this talk, I will discuss some ideas behind the making of an explicit version of the Kuznetsov formula for higher rank groups, which recently led to an explicit Kuznetsov formula for Maaß forms on GSp(4). The talk is based on joint work with Félicien Comtat and Didier Lesesvre.

Speaker: Nathan Ng (University of Lethbridge)

Title: Prime number error terms

Abstract: In 1980 Montgomery made a conjecture on the true order of the error term in the prime number theorem. In the early 1990's Gonek made an analogous conjecture for the sum of the Mobius function. In 2012 I further revised Gonek's conjecture by providing a precise limiting constant. This was based on work on large deviations of sums of independent random variables. Similar ideas can be applied to any prime number error term. In this talk I will speculate about the true order of prime number error terms. Recently, Lamzouri showed that a version of the Linear Independence Conjecture implies part of Montgomery's conjecture. I will show how Lamzouri's argument can be extended to any prime number error term. This talk is based on the preprint `https://arxiv.org/abs/2505.11295`.

Speaker: Byeong-Kweon Oh (Seoul National University)

Title: Isolations of binary quadratic forms from their proper subforms

Abstract: A (positive definite and integral) quadratic form f is called recoverable if any quadratic form representing all proper subforms of f also represents f itself. If there is a quadratic form F representing all subforms of f, except for f itself, then f is called irrecoverable and F is called an isolation of f (from its subforms). In this talk, we find some (ir)recoverable quadratic forms of rank one or two and various isolations of them. This is based on joint work with Jangwon Ju, Daejun Kim, Kyoungmin Kim, and Mingyu Kim.

Speaker: Sudhir Pujahari (National Institute of Science Education and Research (NISER) Bhubaneswar)

Title: Prime scattering geodesic theorem

Abstract: Counting the number of geodesics of a given length on a manifold has long been a central theme in mathematics. In particular, the problem of counting closed geodesics on compact manifolds exhibits a striking analogy with the distribution of prime numbers. In this talk, we explore the distribution of scattering geodesics on the modular surface, focusing on those with prescribed sojourn times. In doing so, we uncover a novel connection between the counting of such scattering geodesics and the arithmetic problem of enumerating positive integers whose prime divisors lie in a fixed arithmetic progression. This is joint work with P.P. Satpathy.

Speaker: Danylo Radchenko (University of Lille)

Title: Multiple polylogarithms and the Steinberg module

Abstract: I will talk about a surprising connection between multiple polylogarithms and the Steinberg module of rationals, and show how this can be used to prove that all multiple polylogarithms of a given weight and depth can be expressed via a single function. I will also discuss implications of this idea for Goncharov's program and conjectures about cohomology of $GL_n(\mathbb{Z})$. The talk is based on a recent joint work with Steven Charlton and Daniil Rudenko.

Speaker: Larry Rolen (Vanderbilt University)

Title: Conjectures of Andrews on partition-theoretic q-series

Abstract: In a famous 1986 paper, Andrews made a number of conjectures on the signs and growth rate of q-series arising from partition theory. Andrews made these based on computer experiments. The first of these functions, the famous function $\sigma(q) := \sum_{n\geq 0} \frac{q^{\frac{n(n+1)}{2}}}{(-q;q)_n}$, had remarkable growth and vanishing behavior which was finally proven by Andrews-Dyson-Hickerson by tying this series to the arithmetic of the field $\mathbb{Q}(\sqrt{6})$. Cohen further uncovered that the numerical phenomenon was due to the q-series being what we would now call, thanks to work of Lewis-Zagier, aperiod integral of a Maass waveform. This was also an early example of the new theory of Zwegers mock Maass theta functions, and of a quantum modular form. In the same paper, Andrews also made conjectures on remarkable sign behavior of partition theoretic functions, such as $v_1(q) := \sum_{n\geq 0} \frac{q^{\frac{n(n+1)}{2}}}{(-q^2;q^2)_n}$. Here, we will discuss recent work, joint with Folsom, Males, and Storzer, establishing some of these. We will also discuss conjectural observations for other questions of Andrews on this function.

Speaker: Jesse Thorner (University of Illinois Urbana-Champaign)

Title: Exceptional zeros of Rankin-Selberg L-functions

Abstract: I will discuss some recent progress on establishing the non-existence of exceptional zeros for certain families of Rankin-Selberg L-functions.

Speaker: Dongxi Ye (Sun Yat-sen University)

Title: Central L-values of congruent number curves

Abstract: In this talk, I will report a recent work on the central L-values of congruent number curves. We shall see that these L-values can be expressed in terms of the square of a CM value of some Dirichlet theta function. This talk is based on joint work with Xuejun Guo and Hongbo Yin.

Speaker: Asif Zaman (University of Toronto)

Title: The least prime with a given factorization type

Abstract: Let K/k be a Galois extension of number fields with Galois group G. For a conjugacy class C of G, the least unramified prime with Frobenius element in C is known to be at most a fixed absolute power of the discriminant of K due to the celebrated work of Lagarias, Montgomery, and Odlyzko (1979). This theorem has been extensively studied with the primary method exploiting statistics of zeros of L-functions. The current record for the exponent is 16 due to Kadiri, Ng, and Wong (2019). For $G = S_n$, I will describe a method based on detecting sign changes that improves this exponent to decay exponentially with n as $n \to \infty$. The ideas also apply to other groups G and certain conjugacy invariant subsets C.

This talk is based on joint work with Peter Cho and Robert Lemke Oliver.