REPRESENTATION THEORY AND NUMBER THEORY CONFERENCE

6 – 8 December, 2023, The University of Hong Kong

Program & Abstracts Conference Schedule

Frontier Lecturers:

George Lusztig (Massachusetts Institute of Technology)

Michael Rapoport (University of Bonn)

Conference Speakers:

Jun Hu (Beijing Institute of Technology)

Ruochuan Liu (Peking University)

Yifeng Liu (Zhejiang University)

Sian Nie (Chinese Academy of Science)

Felix Schremmer (HKU)

Jianyi Shi (East China Normal University)

Toshiaki Shoji (Tongji University)

Binyong Sun (Zhejiang University)

Nanhua Xi (Chinese Academy of Science)

Jie Xiao (Beijing Normal University)

Qingchao Yu (HKU)

Jiping Zhang (Peking University)

Yongchang Zhu (HKUST)

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New Cornerstone Foundation

Representation Theory and Number Theory Conference
December 6 - 8, 2023, The University of Hong Kong

Venue:
Frontiers of Mathematics Lectures in the morning of Dec 6 will be held in T5, Meng Wah Complex, all other Lectures in LG104, KK Leung Building.

	December 6, 2023 (Wednesday)		**December 7, 2023 (Thursday)**	**December 8, 2023 (Friday)**
09:15 - 09:30	Opening Remarks			
09:30 - 10:30	Frontiers of Mathematics Lecture: George Lusztig (T5, Meng Wah Complex)	10:00 - 10:45	Binyong Sun	Ruochuan Liu
10:30 - 11:00	*Refreshments*	10:45 - 11:15	*Coffee Break*	
11:00 - 12:00	Frontiers of Mathematics Lecture: Michael Rapoport (T5, Meng Wah Complex)	11:15 - 12:00	Yongchang Zhu	Yifeng Liu
12:00 - 14:00				
14:00 - 14:45	Nanhua Xi	14:00 - 14:45	Jun Hu	Toshiaki Shoji
14:45 - 15:00	*Break*			
15:00 - 15:45	Sian Nie	15:00 - 15:45	Jianyi Shi	Jie Xiao
15:45 - 16:15	*Coffee Break*			
16:15 - 17:00	Qingchao Yu	16:15 - 17:00	Felix Schremmer	Jiping Zhang

Abstracts

Speaker: Jun Hu (Beijing Institute of Technology)

Title: On the Center Conjecture for the cyclotomic Hecke algebras of type G(r,1,n)

Abstract: I shall report our proof of a longstanding conjecture which asserts that the center of the non-degenerate cyclotomic Hecke algebra of type G(r,1,n) consists of symmetric polynomials in their Jucys-Murphy operators. The proof involves four ingredients: 1) a generalization of Geck-Pfeiffer's work to the cyclotomic Hecke algebra of type G(r,1,n); 2) a passage from cyclotomic Hecke algebra to cyclotomic KLR algebra; 3) a cocenter approach to the Center Conjecture for arbitrary cyclotomic KLR algebras; 4) the seminormal bases theory for semisimple Hecke algebras. Some applications of this conjecture will also be presented. The talk is based on a joint work with Shi Lei.

Speaker: Ruochuan Liu (Peking University)

Title: Congruences between modular forms and geometry of the eigencurve

Abstract: The study of congruences between modular forms is a classic and vital subject in number theory. Hida was the first to establish the theory of p-adic modular forms with slope zero; Hida theory is a core tool in Wiles' proof of the Iwasawa main conjecture, and also one of the main tools in proving Fermat's Last Theorem. Coleman extended Hida's theory to p-adic modular forms of general finite slopes and, together with Mazur, constructed the eigencurve. The geometry of the eigencurve is essential for understanding the congruence properties of modular forms, yet our knowledge about geometry of the eigencurve is very limited. In a recent joint work with Nha Xuan Truong, Xiao Liang and Zhao Bin, we prove the finiteness of irreducible components of the eigencurve under a generic condition.

Speakar: Yifeng Liu (Zhejiang University)

Title: Selmer theta lifts

Abstract: We construct Kudla's generating functions valued in Selmer groups, and prove its modularity under certain conditions. We then define Selmer

theta lifts and compute their p-adic heights, which can be regarded as a p-adic analogue of the arithmetic inner product formula.

Speaker: George Lusztig (Massachusetts Institute of Technology)
Title: The Grothendieck group of unipotent representations; a new basis.
Abstract: The group in the title has a remarkable new basis. We will explain it in the special case of type B,C.

Speaker: Sian Nie (Chinese Academy of Science)
Title: Loop Deligne-Lusztig varieties and Steinberg's cross-sections
Abstract: Loop Deligne-Lusztig varieties (LDLVs for short) were first introduced by Lusztig, whose cohomology are expected to realize interesting reprentations of p-adic groups. For general linear groups, by studying LDLVs of Coxeter type, Boyarchenko, Weinstein, Chan and Ivanov obtained a purely local and geometric realization for a large class of local Langlands and Jaquet-Langlands correspondences. In this talk, we will discuss a generalization (in progress) of their results to other reductive groups.

A key ingredient is a loop group version of Steinberg's cross-sections.

Speaker: Michael Rapoport (University of Bonn)
Title: Fundamental lemma and Arithmetic Fundamental lemma for the whole spherical Hecke algebra.
Abstract: The FL and the AFL for the unit element in the spherical Hecke algebra of the unitary group are recent theorems (of Z. Yun, W. Zhang, R. Beuzart-Plessis, resp. of W. Zhang, A.Mihatsch/W. Zhang, Z. Zhang). Here FL is a statement in p-adic harmonic analysis, whereas AFL is a statement in arithmetic geometry. I will discuss the extension of these statements to an arbitrary element in the spherical Hecke algebra. This is joint work with C. Li and W. Zhang.

Speaker: Felix Schremmer (HKU)
Title: Beyond minute admissible sets
Abstract: If a Shimura variety is fully Hodge-Newton decomposable, its mod p reduction enjoys particularly nice properties. These cases are well-understood and characterized by a certain inequality known as the minute condition.

In a joint project with Xuhua He and Eva Viehmann, we study the next larger cases, where the minute condition is barely not satisfied any more. I will give a classification of these cases and explain the geometric properties of the corresponding Ekedahl-Oort and Kottwitz-Rapoport strata.

Speaker: Jianyi Shi (East China Normal University)
Title: KL-cells in the weighted Coxeter groups
Abstract: Kazhdan-Lusztig cells in the weighted Coxeter groups was first introduced systematically by Lusztig in 2003. He propose a bundle of conjectures on the cells in order to generalize some results from the equal parameter case to the unequal parameter case. In this talk, I intend to give a brief introduction on the topic and report some recent achievements concerning the description of cells.

Speaker: Toshiaki Shoji (Tongji University)
Title : Generalized Green functions of reductive groups
Abstract : Generalized Green functions are important tool for computing irreducible characters of finite reductive groups, introduced by Lusztig. It is known by Luszitg that the computation of generalized Green functions is reduced to the computation of certain functions Y_i, which have a simple expression, up to scalar. In this talk, I will discuss about the problem of determining those scalars. This is a joint work with Frank Luebeck.

Speaker: Binyong Sun (Zhejiang University)
Title: Special unipotent representations of classical Lie groups
Abstract: Inspired by the study of automorphic forms, Arthur suggested

the existence of certain collections of representations of linear real reductive groups. Arthur's desired representations, the special unipotent representations, were defined by Barbasch-Vogan and Adams-Barbasch-Vogan. For classical Lie groups, we construct all the special unipotent representations, by using the theory of local theta correspondence initiated by R. Howe. We also proved that all these representations are unitarizable, as predicted by Arthur-Barbasch-Vogan. This is a report on a joint work with Dan M. Barbarsch, Jia-Jun Ma and Chen-Bo Zhu.

Speaker: Nanhua Xi (Chinese Academy of Science)
Title: Some maximal elements in Weyl module $V((p-2)\rho)$ of $SL_{n+1}(\bar{\mathbb{F}}_p)$
Abstract: We first recall some known results on maximal elements in Weyl modules of $SL_3(\bar{\mathbb{F}}_p)$, then construct maximal elements in some Weyl modules of $SL_4(\bar{\mathbb{F}}_p)$ and some maximal elements in Weyl module $V((p-2)\rho)$ of $SL_{n+1}(\bar{\mathbb{F}}_p)$.

Speaker: Jie Xiao (Beijing Normal University)
Title: Lusztig sheaves and integrable highest weight modules
Abstract: The talk is based on a joint work with J.Fang and Y.Lan. We consider the localization $\mathcal{Q}_{v,w}/\mathcal{N}_v$ of Lusztig's sheaves for framed quivers, and define functors $E_i^{(n)}, F_i^{(n)}, K_i^{\pm}, n \in \mathbb{N}, i \in I$ between the localizations. With these functors, the Grothendieck group of localizations realizes the irreducible integrable highest weight modules $L(\Lambda)$ of quantum groups. Moreover, the nonzero simple perverse sheaves in localizations form the canonical bases of $L(\Lambda)$. We also compare our realization (at $v \to 1$) with Nakajima's realization via quiver varieties and prove that the transition matrix between canonical bases and fundamental classes is upper triangular with diagonal entries all equal to ± 1.

Speaker: Qingchao Yu (HKU)
Title: Zero-dimensional affine Deligne-Lusztig varieties.
Abstract: The affine Deligne-Lusztig variety $X(\mu, b)_K$ arises as the set of geometric points of the underlying reduced scheme of a Rapoport-Zink for-

mal moduli space of p-divisible groups. Its geometric structure is in general very complicated.

It is a natural and interesting question to classify all the quadruple (G, μ, b, K) with dim $X(\mu, b)_K = 0$. This was first asked by Rapoport in 2005, who also made an explicit conjecture in the hyper-special level. In this talk, we will discuss recent progress toward this problem. This is a recent joint work with Xuhua He and Sian Nie.

Speaker: Jiping Zhang (Peking University)
Title: The inductive blockwise Alperin weight conditions
Abstract: Recently, there has been intensive investigation on the Alperin weight conjecture by verifying the inductive blockwise Alperin weight conditions. We will report some of the progress done mainly by our team.

Speaker: Yongchang Zhu (HKUST)
Title: Unitary Representations of Lusztig's $SL(2)$-Monoid over Semifields and Variation Diminishing Operators
Abstract: For a root system G and a semifield F, Lusztig introduced a monoid $G(F)$. In this talk we introduce the unitary representations of $G(F)$ and classified irreducible ones for $SL_2(F)$ for the semifields $\mathbb{R}_{>0}$, $\mathbb{R}^{\text{trop}}$ and $\mathbb{Z}^{\text{trop}}$. All such representations are infinite dimensional except the trivial ones. We show that each irreducible representation admit a model that is totally positive in the sense that all the elements in $SL_2(F)$ act as as variation diminishing operators and preserve the positive functions.