Algebraic Geometry and Complex Manifolds 2004

June 15 – 18, 2004

Room 207, Run Run Shaw Building

Program and Abstracts

Institute of Mathematical Research

The University of Hong Kong

Speakers:

➢ Fabrizio Catanese	Bayreuth, Germany
➢ Meng Chen	Fudan U., China
➢ Koji Cho	Kyushu U., Japan
➢ Alan Huckleberry	Bochum, Germany
➢ Jun-Muk Hwang	KIAS, Korea
➢ Yujiro Kawamata	U. Tokyo, Japan
➢ Masatake Kuranishi	Columbia U., USA
➢ Hing-Sun Luk	CUHK, Hong Kong
➢ Toshiki Mabuchi	Osaka U., Japan
➢ Yoichi Miyaoka	U. Tokyo, Japan
➢ Mihai Paun	Strasbourg, France
➢ Thomas Peternell	Bayreuth, Germany
➢ Georg Schumacher	Marburg, Germany
➢ Bernard Shiffman	Johns Hopkins U., USA
➢ Xiaotao Sun	HKU, Hong Kong
➢ Wing-Keung To	NUS, Singapore
➢ Domingo Toledo	U. Utah, USA
➢ Günther Trautmann	Kaiserslautern, Germany
➢ Hajime Tsuji	Tokyo Inst. Tech., Japan

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Ngaiming Mok, HKU and Yum-Tong Siu, Harvard U. and HKU

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THE UNIVERSITY OF HONG KONG

Institute of Mathematical Research

Algebraic Geometry and Complex Manifolds 2004

June 15 – 18, 2004

Room 207, Run Run Shaw Building

Time / Date	June 15 (Tue)	June 16 (Wed)	June 17 (Thur)	June 18 (Fri)
10:00 – 11:00	Trautmann	Kuranishi	Catanese	Toledo
11:00 – 11:20	*Tea Break*			
11:20 – 12:20	Hwang	Luk	Cho	Huckleberry
Lunch Break				
14:15 – 15:15	Peternell	Schumacher	Kawamata	Mabuchi
15:25 – 16:25	Tsuji	To	Miyaoka	Shiffmann
16:25 – 16:45	*Tea Break*			
16:45 – 17:45	Paun	Sun	Chen	

June 10, 2004

Program

June 15, 2004
Tuesday

9:55 **Ngaiming Mok**, Director, Institute of Mathematical Research
Opening Remarks

Chair: N. Mok, HKU

10:00 – 11:00 **Günther Trautmann**, Kaiserslautern, Germany
Toric sheaves

Tea Break

11:20 – 12:20 **Jun-Muk Hwang**, KIAS, Korea
Chains of minimal rational curves

Lunch Break

Chair: K. Zuo, CUHK

14:15 – 15:15 **Thomas Peternell**, Bayreuth, Germany
On the abundance problem for Kähler manifolds and algebraic varieties

15:25 – 16:25 **Hajime Tsuji**, Tokyo Inst. Tech., Japan
Adjunction property of subvarieties and its applications

Tea Break

16:45 – 17:45 **Mihai Paun**, Strasbourg, France
On n-dimensional compact Kähler manifolds covered by $\mathbf{C}^n$

June 16, 2004
Wednesday

Chair: P.P.W. Wong, HKU

10:00 – 11:00 **Masatake Kuranishi**, Columbia U., USA
Relation between the Bergman and the Szegö kernels

Tea Break

11:20 – 12:20 **Hing-Sun Luk**, CUHK, Hong Kong
Pseudo-Hermitian geometry on real hypersurfaces

Lunch Break

Chair: T.W. Ng, HKU

14:15 – 15:15 **Georg Schumacher**, Marburg, Germany
Petersson-Weil metrics on Douady spaces

15:25 – 16:25 **Wing-Keung To**, NUS, Singapore
The asymptotic behavior of the Takhtajan-Zograf metric

Tea Break

16:45 – 17:45 **Xiaotao Sun**, HKU, Hong Kong
Cohomology of sheaves and differential operators on moduli spaces

June 17, 2004
Thursday

Chair: A. Nicoara, Harvard U.

10:00 – 11:00 **Fabrizio Catanese**, Bayreuth, Germany
Deformation, differentiable and symplectic equivalence for algebraic surfaces

Tea Break

11:20 – 12:20 **Koji Cho**, Kyushu U., Japan
Riemann's quadratic relations of Selberg-type integrals

Lunch Break

Chair: S.-K. Yeung, Purdue U.

14:15 – 15:15 **Yujiro Kawamata**, U. Tokyo, Japan
Birational geometry of derived categories

15:25 – 16:25 **Yoichi Miyaoka**, U. Tokyo, Japan
A note on the Chern classes of stable Higgs bundles on surfaces

Tea Break

16:45 – 17:45 **Meng Chen**, Fudan U., China
Classifying pluricanonical pencils on algebraic threefolds

June 18, 2004
Friday

Chair: G. Heier, Bochum U.

10:00 – 11:00 **Domingo Toledo**, U. Utah, USA
Maps of complex hyperbolic surfaces

Tea Break

11:20 – 12:20 **Alan Huckleberry**, Bochum, Germany
Cycle spaces associated to group actions

Lunch Break

Chair: W.S. Cheung, HKU

14:15 – 15:15 **Toshiki Mabuchi**, Osaka U., Japan
The second fundamental form in the geometry of Kähler potentials, and its application to some uniqueness problem

15:25 – 16:25 **Bernard Shiffman**, Johns Hopkins U., USA
Random algebraic geometry: zeros of fewnomial systems

16:25 **Yum-Tong Siu**, Harvard U. and HKU
Closing Remarks

Abstracts

Fabrizio Catanese, Bayreuth, Germany

Deformation, differentiable and symplectic equivalence for algebraic surfaces

In the talk I will report about joint work with B. Wajnryb. First on the results which we already obtained, showing that even in the case of simply connected minimal algebraic surfaces of general type, deformation and differentiable equivalence do not coincide.

Exhibiting the simple families of abc surfaces which are not deformation equivalent, and proving their diffeomorphism, we get a counterexample to a weaker form of the speculation DEF = DIFF of R. Friedman and J. Morgan, i.e., in the case where (by M. Freedman's theorem) the topological type is completely determined by the numerical invariants of the surface.

The methods of proof are rather general, but if we want to investigate the natural symplectic structures associated to the canonical class, the question of symplectic equivalence becomes more subtle in the 1-connected case. I will explain also work in progress on this second question, especially some beautiful geometry of discriminant curves.

Meng Chen, Fudan U., China

Classifying pluricanonical pencils on algebraic threefolds

Let X be a normal projective 3-fold of general type with at worst $\mathbf{Q}$-factorial terminal singularities. We study the following conjecture: for any integer $m > 2$, $|mK_X|$ is composed with a pencil if any only if $P_m(X) = 2$. We prove that the conjecture is true either for irrational pencils or for m bigger. We also classify pluricanonical pencils for small value of m. There are dozens of supporting examples according to Fletcher and Reid.

Koji Cho, Kyushu U., Japan

Riemann's quadratic relations of Selberg-type integrals

Let $\ell_i = \ell_i(t)$ be an affine linear form in $t = (t_1, \ldots, t_n)$, and a_i a real non-integral constant, for $1 \leq i \leq r$. Put

$$L_i := \{t \in \mathbf{C}^n \mid \ell_i(t) = 0\}, \quad X := \mathbf{C}^n - \cup_i L_i,$$

and consider the (multi-valued) function $u := \prod_{i=1}^r \ell_i(t)^{a_i}$ on X, which defines a local system $\mathcal{L} = \mathcal{L}_u$. Under some generic condition on L_i and a_i, the pair $(X, \mathcal{L})$ is purely n-(co)dimensional, i.e.

$$H_j(X, \mathcal{L}) = 0, \quad H^j(X, \mathcal{L}) = 0, \quad \text{if } j \neq n.$$

We are interested in evaluating the integral

$$\int_{\mathbf{C}^n} |u|^2 \, dt \wedge d\bar{t}, \quad \text{where} \quad dt = dt_1 \wedge \cdots \wedge dt_n$$

in terms of the periods

$$\int_{\gamma_k} u \, dt,$$

where $\{\gamma_k\}$ form a basis of $H_n(X, \mathcal{L})$, and the intersection numbers $\gamma_k \bullet \check{\gamma}_l$, where $\check{\gamma}_k$ is an element of $H_n(X, \check{\mathcal{L}})$ corresponding to $\gamma_k \in H_n(X, \mathcal{L})$. The simplest non-trivial example is

$$\int_{\mathbf{C}} |t|^{2\alpha} |1-t|^{2\beta} \, dt \wedge d\bar{t}.$$

This is well-known to be equal to

$$B(\alpha+1, \beta+1)^2 \, \frac{(1-e^{2\pi i\alpha})(1-e^{2\pi i\beta})}{1-e^{2\pi i(\alpha+\beta)}},$$

where B is the Beta function

$$B(\alpha+1, \beta+1) = \int_0^1 t^\alpha (1-t)^\beta dt.$$

The factor $(1-e^{2\pi i\alpha})(1-e^{2\pi i\beta})/(1-e^{2\pi i(\alpha+\beta)})$ is the reciprocal of the intersection number of the cycles $(0,1) \otimes t^\alpha (1-t)^\beta$ and $(0,1) \otimes t^{-\alpha}(1-t)^{-\beta}$.

In this talk, we give a similar formula for the integral

$$\int_{\mathbf{C}} \prod_{i=1}^r |t - x_i|^{2\alpha_i} \, dt \wedge d\bar{t}.$$

We also consider integrals of Selberg type:

$$\int_{\mathbf{C}^n} \prod_{i=1}^n |t_i|^{2\alpha_i} |1-t_i|^{2\beta_i} \prod_{1 \le i < j \le n} |t_i - t_j|^{2g_{ij}} \, dt \wedge d\bar{t},$$

and

$$\int_{\mathbf{C}^n} \prod_{i=1}^n |t_i|^{2\alpha_i} |1-t_i|^{2\beta} |z_i - t_i|^{2\gamma_i} \prod_{1 \le i < j \le n} |t_i - t_j|^{2g_{ij}} \, dt \wedge d\bar{t}.$$

Alan Huckleberry, Bochum, Germany

Cycle spaces associated to group actions

If G_0 is a real form of a complex semisimple group G, then the G_0-orbits in G-homogeneous rational manifolds provide complex geometric contexts for realization of its representations. Conversely, such orbits and the related representation theory often arise in questions of complex analysis, e.g.,

concerning moduli of complex varieties. In most cases these orbits possess a certain degree of pseudoconcavity, and, in order to shift from the level of cohomology to that of function spaces, one considers associated cycle spaces. Our recent work (joint with J. A. Wolf and with G. Fels) which gives an explicit description of these cycle spaces will be explained in the talk.

Jun-Muk Hwang, KIAS, Korea

Chains of minimal rational curves

Chains of minimal rational curves have been used as an important tool in the study of Fano manifolds. In a joint work with S. Kebekus, we introduce an infinitesimal method to study chains of minimal rational curves via varieties of minimal rational tangents and their higher secants. For many examples of Fano manifolds, this method can be used to compute the minimal length of chains needed to join two general points.

Yujiro Kawamata, U. Tokyo, Japan

Birational geometry of derived categories

I will review results on the conjecture on the equivalence of derived categories for different algebaric varieties which are related by some familiar operations in birational geometry.

Masatake Kuranishi, Columbia U., USA

Relation between the Bergman and the Szegö kernels

We develop the procedure by which we write down the singularity of the Bergmann kernel by means of the singularity of the Szegö kernel in the case of strongly pseudoconvex domains. This a local construction. We use the Fourier integral operatrs.

Hing-Sun Luk, CUHK, Hong Kong

Pseudo-Hermitian geometry on real hypersurfaces

Pseudo-Hermitian geometry on a CR manifold is a reduction of its CR invariant pseudo-conformal geometry by a choice of contact form. We shall discuss some aspects of pseudo-Hermitian geometry. In particular, we shall present a joint work with Song-Ying Li regarding the characterzation of the ball in C^n through the pseudo-Hermitian curvature on the boundary.

Toshiki Mabuchi, Osaka U., Japan

The second fundamental form in the geometry of Kähler potentials, and its application to some uniqueness problem

In this talk, we discuss the notion of second fundamental form in the geometry of Kähler potentials. Let M be a compact complex manifold with a Kähler class κ. For an embedding

$$\iota : N \hookrightarrow M$$

of a compact complex submanifold N into M, let P_M denote the space of all Kähler potentials on M for the class κ, and let P_N denote the space of all Kähler potentials on N for the class $\iota^*\kappa$. Then by pulling back by ι, we have a natural projection of P_M onto P_N. This then allows us to define the second fundamental form for this projection.

For the Chow norm studied by S. Zhang in relation to the stability problem, its seond variation can be actually written in terms of the second fundamental form thus defined, where ι is chosen as the Kodaira embedding of a polarized algebraic manifold (N, L) in the complex projective space $M = \mathbb{P}^*(H^0(N, L^m))$. This then gives us a clear picture in the study of uniqueness, modulo biholomorphisms, of an extremal Kähler metric in a polarization class on a polarized projective algebraic manifold.

Yoichi Miyaoka, U. Tokyo, Japan

A note on the Chern classes of stable Higgs bundles on surfaces

We construct a rank 4 stable Higgs bundle for which the Bogomolov inequality breaks down. From this example, we derive a non-trivial family of stable Higgs bundles of rank 4 with $c_1^2 = c_2 = 0$. Simpson's theorem implies that this gives a non-trivial deformation of indecomposable representations of the fundamental group of the base surface.

Mihai Paun, Strasbourg, France

On n-dimensional compact Kähler manifolds covered by $\mathbf{C}^n$

We report on a joint work in progress with F. Campana. Consider X a compact Kähler manifold, such that its canonical is nef. Assume also that there exist a holomorphic map from $\mathbf{C}^n$ to X, whose Jacobian is not identically zero. We prove that in some cases such a manifold has numerically trivial canonical bundle.

Thomas Peternell, Bayreuth, Germany

On the abundance problem for Kähler manifolds and algebraic varieties

I report on joint work with S. Boucksom, J.P. Demailly and M. Paun. In particular I discuss consequences of the following result characterising pseudo-effective line bundles on a projective manifold. Namely a line bundle is pseudo-effective if and only if its degree on any member of a covering family of curves is non-negative. As a corollary, a projective manifold is uniruled if

and only if its canonical bundle is not pseudo-effective. The applications concern (a part of) the abundance problem on projective 4-folds: a 4-fold with pseudo-effective canonical bundle, with the additional property that it is zero on some covering family of curves, has positive Kodaira dimension. I also discuss the present state of the abundance problem of Kähler threefolds.

Georg Schumacher, Marburg, Germany

Petersson-Weil metrics on Douady spaces

We address the problems of degeneration and curvature. Previously, for moduli of smooth hypersurfaces, a generalized Petersson-Weil metric was introduced, using complete Kaehler-Einstein metrics on the complements. The curvature tensor was computed explicitly, and hyperbolicity followed. For general dimensions, we use a different approach. We compute the asymptotic behavior, show that the Petersson-Weil metric is the curvature of the Quillen metric for certain determinant bundles, and investigate the situation with respect to degenerations. A modification of the Petersson-Weil metric yields a certain hermitian metric on the moduli space, whose curvature is computed.

Bernard Shiffman, Johns Hopkins U., USA

Random algebraic geometry: zeros of fewnomial systems

We discuss the distribution of zeros of systems of random complex fewnomials. We choose the exponents as well as the coefficients at random. We obtain asymptotic formulas for the distribution of zeros as the degrees tend to infinity and we show that the zeros are self-averaging in the sense that typical systems are close to the average behavior for large degrees.

Xiaotao Sun, HKU, Hong Kong

Cohomology of sheaves and differential operators on moduli spaces

The talk is on a joint work with I-Hsun Tsai. We prove an identification theorem between the first direct image of sheaves on curves and the sheaves of differential operators on moduli spaces of vector bundles on the curves. As an application, we give a new construction of Hitchin's connection on the moduli space of curves with level structures, which has a natural extension to the boundary of certain singular stable curves.

Wing-Keung To, NUS, Singapore

The asymptotic behavior of the Takhtajan-Zograf metric

The Takhtajan-Zograf metric is a metric defined on the Teichmüller space for Riemann surfaces of type (g, n) with $n > 0$. Similar to the Weil-Petersson metric, the Takhtajan-Zograf metric is an

incomplete Kähler metric invariant under the Teichmüller modular group.
In this talk, I will describe a recent joint work with K. Obitsu and L. Weng on the asymptotic behavior of this metric.

Domingo Toledo, U. Utah, USA
Maps of complex hyperbolic surfaces

We study a class of examples of surjective holomorphic maps between compact two-dimensional ball quotients that are not covering maps. We find all such maps that can be written in terms of hypergeometric functions. They have the property that the induced homomorphism in fundamental groups is not injective. They include Mostow's example of a non-injective homomorphism, and are motivated by that example. We study the singularity behavior of these maps, and state a number of natural problems that are suggested by these examples.

Günther Trautmann, Kaiserslautern, Germany
Toric sheaves

A toric sheaf is a torus invariant coherent sheaf on a toric variety. Any such sheaf is related to a fine-graded and finitely generated module over the Cox coordinate ring of the variety. A report on recent results on the structure of toric sheaves is given, stressing global primary decompositions and torus invariant resolutions. The existence of a global primary decomposition of an equivariant sheaf is proved in the more general setting for quasi-homogeneous varieties, and is a refinement of an early theorem of Yum-tong Siu.

Hajime Tsuji, Tokyo Inst. Tech., Japan
Adjunction property of subvarieties and its applications

It is important to know the relation between the canonical ring of the subvariety and that of the ambient space. In this talk, I would like to talk about the adjunction properties of subvarieties and its application.