Holomorphic Vector Fields on Fano Manifolds

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Abstract

Let X be a projective uniruled manifold of dimension n. We consider the Lie algebra of holomorphic vector fields $\Gamma(X) := \Gamma(X, T_X)$ on X. Jun-Muk Hwang had earlier established an estimate on $\gamma(X) := dim(\Gamma(X))$ depending only on n under the assumption that X is of Picard number 1. The bound is exponential. When X is of Picard number > 1 by considering Hirzebruch surfaces one sees that there are no estimates depending only on dimensions.

As a component in a joint project with Jun-Muk Hwang to study Fano manifolds in terms of the geometry of their spaces of rational curves, we consider for Fano manifolds X of Picard number 1 the question of sharp bounds on vanishing orders of holomorphic vector fields and on $\gamma(X)$. The intervening geometric object is the variety of minimal rational tangents (VMRT), which is the collection at a generic point of the set of all tangents to minimal rational curves. Under certain geometric assumptions on VMRTs we show that at a generic point of X there is no nontrivial holomorphic vector fields vanishing to the order ≥ 3, and, with a slightly stronger hypothesis we show that $\gamma(X) \leq n^2 + 2n$, where equality holds only if X is the projective space.

One main motivation is to study the question of deformation rigidity of certain Fano manifolds of Picard number 1 as projective manifolds especially those which are rational homogeneous manifolds. The method of holomorphic vector fields leads indeed to a proof of deformation rigidity in this context in the remaining difficult cases, e..g., when the model space is the Grassmannian of isotropic k-dimensional vector subspaces in a 2n-dimensional symplectic vector spaces with $n > k$.