The Unique Embedding and Tietze Transformation

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Abstract

A polynomial p in a polynomial or free associative algebra A over a field K (of characteristic zero) admits the unique embedding in A, if for any polynomial q in $A, A/ < p >$ is K-isomorphic to $A/ < q >$ implies that there exists a K-automorphism of A taking p to q. The general Embedding Problem asks for a given polynomial p, whether p admits the unique embedding. In general the Embedding Problem has a negative solution. When p is a coordinate (free generatorgenerator) of A, the problem becomes the Embedding Conjecture of Abhyankar-Sathaye in case A is a polynomial algebra. For a free associative algebra A (i.e. noncommutative polynomial algebra), the corresponding Conjecture is well-known as a Problem of George Bergman.

When A is the polynomial algebra of rank 2, the Conjecture was solved by Abhyankar-Moh-Suzuki, but for rank more than 2, the Conjecture is still open. When A is the free associative algebra of any rank, the Conjecture (Bergman's problem) was completely solved by Vladimir Shpilrain and the speaker. In this talk, Tietze transformtion, an approach to the Embedding Conjecture introduced by the speaker and his collaborator, will be explored. Some related topics, such as the Lifting Problem and the Tame Generator Conjecture, will be discussed. Among other things, the solution of Begman's Problem will be explained.

This talk is motivated by a recent question in affine Hecke algebra raised by Xi Nanhua.