Dr. Sachiko Hamano  
Osaka City University, Japan

Variation of the $\mathbf{a}$-span of an open Riemann surface and pseudoconvexity

10:00 – 11:00 am

Let $R$ be a homologically marked open Riemann surface of genus $g$ ($1 \leq g < \infty$) with $C^\infty$ smooth boundary. If there exists a conformal embedding of $R$ into a marked closed Riemann surface $S$ of the same genus, then a closing $S$ of $R$ induces the Riemann’s period matrix $(\tau_{kj})_{1\leq j,k \leq g}$. M. Shiba showed that, for any real $g$-vector $\mathbf{a} := (a_1, \ldots, a_g) \neq \mathbf{0}$ and any closing $S$ of $R$, the $\mathbf{a}$-modulus

$$\tau_\mathbf{a} := \sum_{j,k=1}^g a_j a_k \tau_{kj}$$

is a closed disk $\{\tau_\mathbf{a} \in \mathbb{C} | |\tau_\mathbf{a} - \tau_\mathbf{a}^*| \leq \rho_\mathbf{a}\} \subset \mathbb{H}$. Here, $\tau_\mathbf{a}^* \in \mathbb{H}$ and $\rho_\mathbf{a} \in \mathbb{R}$ with $0 \leq \rho_\mathbf{a} < \text{Im} \tau_\mathbf{a}^*$, which depend only on $\mathbf{a}$ and $R$. We call $\rho_\mathbf{a}$ the $\mathbf{a}$-span of $R$. In this talk, we establish the variational formula of the $\mathbf{a}$-span $\rho_\mathbf{a}(t)$ for the deforming open Riemann surface $R(t)$ of genus $g$ with complex parameter $t \in \Delta := \{|t| < r\} \subset \mathbb{C}$, and show that, if the total space $\pi : \mathcal{R} \to \Delta$ of $\pi^{-1}(t) = R(t)$ is a two-dimensional pseudoconvex domain, then $\rho_\mathbf{a}(t)$ is subharmonic on $\Delta$. This talk is based on a joint work with Masakazu Shiba and Hiroshi Yamaguchi.

11:00 – 11:20 am  
Tea Break

Dr. Weiyu Colin Tan  
National University of Singapore

Archimedean Positivstellensätze

11:20 am – 12:20 pm

Berr-Wörmann showed that Pólya’s Positivstellensatz for forms relative to the simplex is a consequence of Krivine’s Representation Theorem. Krivine’s result is a sufficient condition for an element of a real algebra $A$ to lie in an archimedean subsemialgebra $S$ over the nonnegative reals. We state a more general sufficient condition for an element of an arbitrary $A$-module to lie in a $S$-semimodule that admits an order unit. With this generalized Representation Theorem, we give uniform proofs of all other known toric archimedean Positivstellensätze.

This meeting is hosted by the Institute of Mathematical Research, HKU.

All are Welcome