Workshop on Complex Geometry

July 10 - 12, 2019
Room 210, Run Run Shaw Bldg., HKU

Program and Abstracts

Institute of Mathematical Research
Department of Mathematics
### Speakers:

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<td>Baohua Fu</td>
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<td>Jaehyun Hong</td>
<td>KIAS, Seoul, Korea</td>
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<td>Jun-Muk Hwang</td>
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<td>Sui-Chung Ng</td>
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<td>Yum-Tong Siu</td>
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### Organizers: Ngaiming Mok & Tuen Wai Ng

*Email: imr@maths.hku.hk*
## Workshop on Complex Geometry

### July 10 - 12, 2019

Room 210, Run Run Shaw Bldg., HKU

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<td>10:00 – 11:00</td>
<td>Siu</td>
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<td>Tea Break</td>
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<td>11:20 – 12:20</td>
<td>To</td>
<td>Fu</td>
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<td>14:30 – 15:30</td>
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_Last updated: Jul 2, 2019_
9:55  Ngaiming Mok, Director, Institute of Mathematical Research
Opening Remarks

10:00 – 11:00  Yum-Tong Siu, Harvard U., USA
Vanishing Orders of Jacobian Determinants and Metrics from Sections of Line Bundles

Tea Break

11:20 – 12:20  Wing-Keung To, National U. of Singapore
Volume estimates, singular potentials and applications

Lunch Break

14:30 – 15:30  Sai-Kee Yeung, Purdue U., USA
Torelli locus and symmetric subvarieties of a Siegel moduli variety

Tea Break
10:00 – 11:00  **Jun-Muk Hwang**, KIAS, Seoul, Korea
*On Hirschowitz’s conjecture on the formal principle*

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**Tea Break**

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11:20 – 12:20  **Baohua Fu**, The Chinese Academy of Sciences
*Geometry of linear sections of rational homogeneous spaces*

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**Lunch Break**

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14:30 – 15:30  **Xiaotao Sun**, Tianjin U.
*Slope inequality and Miyaoka-Yau type inequality*

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**Tea Break**

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15:50 – 16:50  **Jaehyun Hong**, KIAS, Seoul, Korea
*Cohomology of Hessenberg varieties*
10:00 – 11:00  **Xiangyu Zhou**, The Chinese Academy of Sciences  
Multiplier ideal sheaves: properties and applications

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Tea Break

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11:20 – 12:20  **Sui-Chung Ng**, ECNU, Shanghai  
Syzygies of ample line bundles on fake projective planes

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Lunch Break

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14:30 – 15:30  **Tuen-Wai Ng**, HKU, Hong Kong  
The Squeezing Functions and Fridman Functions on Planar Domains

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Tea Break

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15:50 – 16:50  **Yik-Man Chiang**, HKUST, Hong Kong  
Degenerated biconfluent Heun operator and Painlevé IV
Abstracts

Yik-Man Chiang, HKUST, Hong Kong

Degenerated biconfluent Heun operator and Painlevé IV

It is classical that the Painlevé equations describe integral curves on moduli spaces of Fuchsian connections with four punctures on the Riemann Sphere as well as their confluent counterparts. These Painlevé equations also exhibit certain affine Weyl group symmetries with respect to their parameters in which their solutions degenerate. We exhibit that one can identify the parameter space for Painlevé IV directly from the biconfluent Heun connection with degenerate monodromy/Stokes data. Moreover, explicit horizontal sections of certain biconfluent Heun connections are derived. These are joint works with A. Ching, C. K. Law, C. Y. Tsang and G. F. Yu.

Baohua Fu, The Chinese Academy of Sciences

Geometry of linear sections of rational homogeneous spaces

Rational homogeneous varieties are among the simplest algebraic varieties, and a better understanding of them is always a motivation for the development of algebraic geometry. The geometry of general linear sections of rational homogeneous spaces are as expected, but special ones may have much richer geometry which remains to be explored systematically. I’ll survey recent progress in this direction.

Jaehyun Hong, KIAS, Seoul, Korea

Cohomology of Hessenberg varieties

Hessenberg varieties are subvarieties of complete flag varieties $G/B$ associated to Hessenberg subspaces, which are $b$-submodules of $g$ containing $b$, where $b$ and $g$ are the Lie algebras of $B$ and $G$. When the subspace is the sum of $b$ and the negative root spaces, it is the toric variety associated with the fan defined by the hyperplanes orthogonal to the roots of $g$. In this talk we will investigate the Weyl group action on the cohomology spaces of Hessenberg varieties.

Jun-Muk Hwang, KIAS, Seoul, Korea

On Hirschowitz’s conjecture on the formal principle

A compact complex submanifold of a complex manifold is said to satisfy the formal principle if its formal neighborhood determines its germ of analytic neighborhoods. In 1981, Hirschowitz conjectured that an unobstructed submanifold satisfies the formal principle if its normal bundle is globally generated. I explain an approach to this conjecture by Cartan’s equivalence method for geometric structures.
Sui-Chung Ng, ECNU, Shanghai

*Syzygies of ample line bundles on fake projective planes*

We will talk about the syzygies of the embeddings defined by the ample line bundles on a fake projective plane. The syzygies are studied in terms of the so-called $N_p$ properties, in which for instance the $N_0$ property is equivalent to projective normality and the $N_1$ property is the same as projective normality together with the generation of vanishing ideal by quadratic polynomials. We will give an explicit lower bound $M_p$ such that $L^m$ will satisfy the $N_p$ property for any ample line bundle $L$ on a fake projective plane and when $m$ is at least $M_p$. This is a joint work with Sai-Kee Yeung.

Tuen Wai Ng, HKU, Hong Kong

*The Squeezing Functions and Fridman Functions on Planar Domains*

In 2012, Deng, Guan and Zhang introduced the concept of squeezing function for a bounded domain. This function of several complex variables is a very interesting new biholomorphic invariant and it also has a dual function, namely the Fridman function. In this talk, we shall focus on the squeezing functions and Fridman functions of some simple planar domains and explain how some classical tools in geometric function theory can be used to study these functions. The results are joint work with Chiu Chak Tang and Jonathan Tsai.

Yum-Tong Siu, Harvard U., USA

*Vanishing Orders of Jacobian Determinants and Metrics from Sections of Line Bundles*

The vanishing orders of multiplier ideal sheaves take over the role of PDE estimates in problems of complex geometry. There are two important techniques involving such vanishing orders. One is the method of differential relations to use Jacobian determinants to reduce vanishing orders, which originated from complex Neumann problems of weakly pseudoconvex domains. Another is the method of precise achievement of vanishing orders of metrics from sections of line bundles, which is the foundation for analytic approaches to effective problems and complex geometry problems involving plurigenera, the finite generation of the canonical ring and the abundance conjecture.

Xiaotao Sun, Tianjin U.

*Slope inequality and Miyaoka-Yau type inequality*

Let $S$ be a minimal smooth projective surface of general type over an algebraically closed field of characteristic $p > 0$. In this talk (joint with Yi Gu and Mingshuo Zhou), I will show a slope inequality and its corollary: a Miyaoka-Yau type inequality. In particular, it implies that $\chi(O_S) > 0$ for any smooth surfaces $S$ of general type (a question posed by Shepherd-Barron in 1991).

Wing-Keung To, National U. Singapore

*Volume estimates, singular potentials and applications*

In this talk, I will discuss some joint works with J.-M. Hwang on finding sharp lower bounds on volumes of certain complex analytic varieties via singular potentials. The background manifolds are bounded
symmetric domains and their quotients. Some applications/motivations will also be discussed.

**Sai-Kee Yeung**, Purdue U., USA

*Torelli locus and symmetric subvarieties of a Siegel moduli variety*

The purpose of this talk is to present some geometric approaches to study the possibility of realization of a locally symmetric space as a totally geodesic subvariety in the Torelli locus of a Siegel modular space. Restricting to Hermitian symmetric subvarieties of a Siegel modular variety, it is related to conjectures of Coleman and Oort for cases other than complex curves.

**Xiangyu Zhou**, The Chinese Academy of Sciences

*Multiplier ideal sheaves: properties and applications*

After recalling basic properties of the multiplier ideal sheaves, we talk about some new properties and applications.