



*Institute of Mathematical Research
Department of Mathematics*

Workshop on Geometry and Lie Theory

June 17 - 19, 2019

Room 210, Run Run Shaw Building, HKU

Monday, June 17	
11:00 – 12:00	Jihyeon Jessie Yang , Marian University, USA <i>Talk 1: Introduction to Tropical geometry and Newton-Okounkov theory</i>
<i>Lunch Break</i>	
14:00 – 15:00	Jihyeon Jessie Yang , Marian University, USA <i>Talk 2: Tropical geometry and Newton-Okounkov cones for Grassmannian of planes from compactifications</i>
<i>Coffee Break</i>	
15:30 - 16:30	Xiaomeng Xu , MIT, USA <i>Stokes phenomenon and integrable systems</i>
Tuesday, June 18	
10:00 - 11:00	Daping Weng , Michigan State University, USA <i>Introduction to cluster ensembles and the cluster duality conjecture (Lecture 1)</i>
<i>Coffee Break</i>	
11:30 – 12:30	Daping Weng , Michigan State University, USA <i>Introduction to cluster ensembles and the cluster duality conjecture (Lecture 2)</i>
<i>Lunch Break</i>	
14:30 – 15:30	Benjamin Hoffman , Cornell University, USA <i>String domains for coadjoint orbits</i>
<i>Coffee Break</i>	
16:00 – 17:00	Reyer Sjamaar , Cornell University, USA <i>The convexity theorem for Hamiltonian actions on conformal symplectic manifolds</i>
<i>Workshop Dinner</i>	
Wednesday, June 19	
16:00 – 17:00	Daping Weng , Michigan State University, USA <i>Cluster Structures on Double Bott-Samelson Cells</i>

Titles & Abstracts

Workshop on Geometry and Lie Theory June 17 - 19, 2019, HKU

Jihyeon Jessie Yang, Marian University, USA

Talk 1: Introduction to Tropical geometry and Newton-Okounkov theory

I will introduce two developments in (polyhedral) combinatorics in the approach to solve problems in algebraic geometry and representation theory. They are Tropical geometry and Newton-Okounkov theory. I will talk about the following three questions:

1. What are they? : Find an explicit description of each combinatorial object.
2. How to use them? : Find applications
3. How are they related? : Find relations between these two theories.

Talk 2: Tropical geometry and Newton-Okounkov cones for Grassmannian of planes from compactifications

I will revisit the last question in the previous talk, “How are they related?”, for the case of Grassmannian of 2-planes. We construct a family of compactifications of the affine cone of the Grassmannian of 2-planes. We show that both the Tropical and Newton-Okounkov results can be recovered from these compactifications

Xiaomeng Xu, MIT, USA

Stokes phenomenon and integrable systems

This talk includes an introduction to the Stokes phenomenon of differential equations, from which we show that various structures (including new ones) in integrable systems, Poisson geometry and representation theory naturally arises.

Daping Weng, Michigan State University, USA

Introduction to cluster ensembles and the cluster duality conjecture (Lecture 1 & 2)

In early 2000s, Fock and Goncharov introduced cluster varieties as geometric counterparts of Fomin and Zelevinskys cluster algebras, and they formulated an important conjecture in cluster theory called the cluster duality conjecture. The cluster duality conjecture asserts the existence of certain canonical bases for the algebras of regular functions on pairs of cluster varieties. In 2014, Gross, Hacking, Keel, and Kontsevich proved a weak version of the cluster duality conjecture and gave a sufficient condition for the full cluster duality conjecture. This sufficient condition is implied by the existence of cluster Donaldson-Thomas transformation defined by Goncharov and Shen. In these two lectures I will give an introduction to the relevant topics and state some of the known results.

Benjamin Hoffman, Cornell University, USA

String domains for coadjoint orbits

For each regular coadjoint orbit of a compact group, we construct an exhaustion by symplectic embeddings of toric domains. As a by-product we arrive at a conjectured formula for the Gromov width of coadjoint orbits. Our method combines ideas from Poisson-Lie groups and from the geometric crystals of Berenstein-Kazhdan. We also prove similar results for multiplicity-free spaces. This is joint work with A. Alekseev, J. Lane, and Y. Li.

Reyer Sjamaar, Cornell University, USA

The convexity theorem for Hamiltonian actions on conformal symplectic manifolds

Consider a Hamiltonian action of a compact connected Lie group on a conformal symplectic manifold. We prove a convexity theorem for the moment map under the assumption that the action is of Lee type, which establishes an analog of Kirwans convexity theorem in conformal symplectic geometry. This is joint work with Youming Chen and Xiangdong Yang.

Daping Weng, Michigan State University, USA

Cluster Structures on Double Bott-Samelson Cells

Let G be a Kac-Peterson group associated to a symmetrizable generalized Cartan matrix. Let (b, d) be a pair of positive braids associated to the root system. We define the double Bott-Samelson cell associated to G and (b, d) to be the moduli space of configurations of flags satisfying certain relative position conditions. We prove that they are affine varieties and their coordinate rings are upper cluster algebras. We construct the Donaldson-Thomas transformation on double Bott-Samelson cells and show that it is a cluster transformation. In the cases where G is semisimple and the positive braid (b, d) satisfies a certain condition, we prove a periodicity result of the Donaldson-Thomas transformation, and as an application of our periodicity result, we obtain a new geometric proof of Zamolodchikov's periodicity conjecture in the cases of $D \otimes A_n$. This is joint work with Linhui Shen.