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Cluster Structures on Double Bott-Samelson Cells

Let G be a Kac-Peterson group associated to a symmetrizable generalized Cartan matrix. Let (b, d) be a pair of positive braids associated to the root system. We define the double Bott-Samelson cell associated to G and (b, d) to be the moduli space of configurations of flags satisfying certain relative position conditions. We prove that they are affine varieties and their coordinate rings are upper cluster algebras. We construct the Donaldson-Thomas transformation on double Bott-Samelson cells and show that it is a cluster transformation. In the cases where G is semisimple and the positive braid (b, d) satisfies a certain condition, we prove a periodicity result of the Donaldson-Thomas transformation, and as an application of our periodicity result, we obtain a new geometric proof of Zamolodchikov's periodicity conjecture in the cases of $D \otimes A_n$. This is joint work with Linhui Shen.