Chapter 6

Official Curriculum in Mathematics in Ancient China: How did Candidates Study for the Examination?

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This chapter starts with a brief general account of mathematics education in ancient China, then discusses in detail the official curriculum and the state examination system in mathematics in the Tang Dynasty. In the second part of the chapter some examples of examination questions are re-constructed with "circumstantial evidence" to offer an alternative viewpoint from a traditional one, to argue that study in mathematics in ancient China did not proceed in an examination-oriented, rote-based learning environment. This "animated" historical account may help to shed some light on the comparative study of mathematics education in the East and West.

Key words: mathematics curriculum, state examination system, Tang Dynasty

1 Introduction: The CHC Learner Paradox and the CHC Teacher Paradox

Ever since the early 1990s some educators have begun to pay attention to cultural differences that may affect the learning and teaching of subjects like science and mathematics, which are usually regarded as universal in content (Cai, 1995; Stevenson & Stigler, 1992; Watkins & Biggs, 1996). Interest in this aspect is further reinforced by the results coming out of several international studies sponsored by organizations like the International Association for the Study of Educational Achievement (IEA) or the Organization for Economic Cooperation and Development

(OECD). In particular, the learning process of Asian students brought up in the tradition of the Confucian-heritage culture (CHC) has become a much discussed issue in the past decade (Leung, 2001; Watkins & Biggs, 1996; Wong, 1998). As a natural consequence, the teaching process of Asian teachers in CHC classrooms has come under review as well (Ma, 1999; Stigler & Hiebert, 1999; Watkins & Biggs, 2001). These two closely inter-related issues can be brought into focus in the form of two paradoxes, *viz*

(1) The CHC Learner Paradox: CHC students are perceived as using low-level, rote-based strategies in a classroom environment which should not be conducive to high achievement, yet CHC students report a preference for high-level, meaning-based learning strategies and they achieve significantly better in international assessments!
(2) The CHC Teacher Paradox: Teachers in CHC classrooms produce a positive learning outcome under substandard conditions that Western educators would regard as most unpromising!

In this chapter we look at these issues from a historical angle by investigating the official curriculum in mathematics in ancient China. After giving a brief general account of mathematics education in ancient China, we will confine the discussion to the curriculum of the state university of the Tang Dynasty (唐, 618 – 907), particularly to the state examination system in mathematics in that period. That period is chosen not only because the state examination system in mathematics was in its most established form by then, but because the system in later dynasties was either modeled after it or was no longer in place. A major part of this chapter is spent on the state examination system, because the CHC classroom is usually labeled as dominated by an examination-oriented culture and it is commonly believed that an examination-oriented culture hinders the learning process. But is it really so? By piecing together, from official records in the ancient chronicles, a "rational re-construction" of the state examination in mathematics in the Tang Dynasty — as there is no single extant document on the examination itself — we are prepared to ask questions such as: Is the examination system really that damaging to learning? Is the examination a necessary evil? Or is it beneficial to the learning process to some extent? Was the state examination a mere test on rote learning?

The author borrows heavily from three of his articles (Siu, 1995, 2001; Siu & Volkov, 1999), which happen to be not very readily accessible to teachers and educators in mathematics, so that an edited summarized presentation here may prove useful to shed light on a comparative study of mathematics education in the East and West. The first article, which is the text of a talk given in 1992 on mathematics education in ancient China, should be regarded as a "first approximation" in view of the more in-depth historical study carried out later (in collaboration with the historian of mathematics Alexeï Volkov) in the second article. The third article, which is the text of a talk given in 1998, is more inclined towards the pedagogical aspect and is closest to the main message to be conveyed in this chapter. The author wishes to acknowledge with heartfelt gratitude the permission of Madame Patricia Radelet-de Grave, the editor of the Proceedings of the Third European Summer University held at Louvain-la-Neuve and Leuven in 1998, for him to incorporate parts of his article (Siu, 2001) into Section 4, Section 5 and Section 6 of this chapter.

2 Mathematics Education in Ancient China

Even if it is debatable what constituted a genuine beginning of mathematics in the history of civilization — is it drawing? or counting? or calculation? or argumentation? or reasoning? or proof? — it seems fair to say that mathematics education, at least in its narrower sense of transmission of mathematical skill and knowledge, came into existence alongside mathematics.

In ancient China the school system in its formal setting began in about 2000 B.C. during the latter part of the Xia Dynasty (夏, 21st century B.C. to 16th century B.C.). Run by the state, it was intended as a training ground for youths and children of the aristocracy. The official system became more institutionalized in the Shang Dynasty (商, 16th century B.C. to 1066 B.C.) and the Western Zhou Dynasty (西周, 1066 B.C. to 771 B.C.). The invasion of barbarian hordes in 770 B.C. forced a transfer of the capital, thereby starting the Eastern Zhou Dynasty (东周, 770 B.C. to 256 B.C.) during which the state power of the Zhou Kingdom was waning, resulting in a continual vying for political domination among the many feudal lords. This long period known as the "Spring and Autumn Period" (春秋时代) followed by the "Warring States Period" (战国时代) was to last for five centuries. Beset with conflicts and unrest, it was a stirring and eventful period as well as, ironically, a most stimulating and prosperous period in terms of intellectual development in the history of China. The decline in state-run institutions of learning was more than compensated for by the formation of private academies (not necessarily with a physical setting) around some scholars of fame. In later dynasties such private academics gradually developed into an important part of the education system (with physical setting) under the name of *shu yuan* (书院, an academy of classical learning). 'An academy of classical learning', was originally set up as an official organization in the Tang Dynasty to collect and compile books of learning. The evolvement of 'an academy of classical learning' has been a topic of intensive research. However, as there is hardly any trace of a mathematics curriculum found in documents about these private academics, we will not further dwell on them, but note that this dual system of learning, which comprised state-run institutions and private academies side by side, persisted in China for the next two millennia (Chen & Deng, 1997; Ding & Liu, 1992; Zhao & Xue, 1995; Zhang, 1985).

In the Han Dynasty (汉, 206 B.C. to 220 A.D.) Confucianism was established as the supreme state philosophy. With the emphasis on the study of classics, mathematics was accorded attention after a fashion, because some classics made allusions to mathematical knowledge here and there. Indeed, for the curriculum in higher education, the "Six Arts" (六艺) comprised Rituals (礼), Music (乐), Archery (射), Charioteering (御), History (书) and Arithmetic (数). (In the early days, the subject Arithmetic was intimately tied up with numerology. The latter was referred to as the "internal arithmetic" (内算), while what we understand as mathematics today was referred to as "external arithmetic" (外算)

(Liu, 1993, p.71).) This sixth Art of Arithmetic was further divided into nine topics, as recorded in the commentary by ZHENG Xuan (郑玄) in the 2nd century, with their titles not too much different from the nine chapter titles of the very famous mathematical classics *jiu zhang suan shu* (九章算术, Nine Chapters on the Mathematical Art), which is believed to have been compiled some time between 100 B.C. and 100 A.D. In 1984 in an Han tomb in the Hubei Province a book written on bamboo strips bearing the title *suan shu shu* (算数书, Book on the Mathematical Art) was discovered (during excavation). The book is dated at around 200 B.C. and its content exhibits a marked resemblance to that of 'Nine Chapters on the Mathematical Art', lending evidence to the belief that the content of 'Nine Chapters on the Mathematical Art' is much older than the book itself (Peng, 2001). In any case, the format of either book became a prototype for all Chinese mathematical classics in the subsequent one-and-a-half millennia. 'Nine Chapters on the Mathematical Art' is a conglomeration of 246 mathematical problems grouped into nine chapters: (1) Survey of land, (2) Millet and rice, (3) Distribution by progressions, (4) Diminishing breadth, (5) Consultation on engineering works, (6) Imperial taxation, (7) Excess and deficiency, (8) Calculating by tabulation, (9) *gou-gu* (勾股，right triangles). In the text a few problems of the same type are given, along with answers, after which a general method (algorithm) follows. It should be noted that the numerical data given in the text are specific rather than special, so they are in fact generic, making the method (algorithm) essentially a general procedure. In the very early edition no further explanation was added to the text, that being perhaps supplied by the teachers. Later editions were appended with commentaries from various authors, which were an indication of serious and assiduous self-study on the part of the author and provided useful aid-to-study for future generations of readers. One of the most notable commentators, LIU Hui (刘徽) of the mid 3rd century, wrote in the preface, "I studied 'Nine Chapters on the Mathematical Art' at an early age and perused it when I got older. I see the separation of the Yin and the Yang and arrive at the root of the mathematical art. In this process of probing I comprehend its meaning. Despite ignorance and incompetence on my part I dare expose what I understand in these commentaries. Things are related to each other through logical reasoning so that like branches of a tree, diversified as they are, they nevertheless come out of a single trunk. If we elucidated by prose and illustrated by pictures, then we may be able to attain conciseness as well as comprehensiveness, clarity as well as rigor. Looking at a part we will understand the rest." (Siu, 1993, p.355). This is a clear message of the balanced employment of rigorous argument and heuristic reasoning with an aim of achieving enhanced understanding. For more illustrative examples, readers can consult (Siu, 1993).

Beginning with the Sui Dynasty (隋, 581 – 618), a comprehensive official system of education was established, further consolidated in the Tang Dynasty (唐, 681 – 907) and the Song Dynasty (宋, 960 – 1279). There was a well-planned curriculum, including the syllabus and the adopted textbooks, for each of several chosen disciplines. The institutional setting for these chosen disciplines was documented down to the quota of student enrollment, the number of the teaching and administrative staff, and the criteria for admission. State examinations for these chosen disciplines were held regularly and successful candidates were appointed to official posts according to merit in their performance at examinations. As explained in Section 1 of this chapter we will confine our attention only to the discipline of mathematics as recorded in chronicles about the official system in the Tang Dynasty. This will be discussed in Section 3 and 5.

Although the official system of education was furthered consolidated and expanded in the Song Dynasty, in the discipline of mathematics there was, however, a decline, with the exception of a strengthening of curriculum in calendarial reckoning and astronomy/astrology. Subsequently, mathematics was even removed as a subject altogether from the state examinations, and was never reinstated in the next several dynasties. From the beginning of the 17^{th} century onwards, mathematical development in China began to come under foreign influence through large-scale contact with Western mathematics, first during the late Ming Dynasty (明, 1368 – 1644), then during the early Qing Dynasty (清, 1616 – 1911) and again during the final quarter of the Qing Dynasty in the mid 19^{th} century. As Chinese mathematics entered its modern era and gradually fused with a more universal mathematics (universal in the sense that it is practiced and studied along a certain trend and style in countries which play a dominant role in world politics and in cultural influence), mathematics education in China became basically not too different from that of most other (Western) countries. (For more references on mathematics education in ancient China, see (Chen, 2002; Ding & Zhang, 1989; Jin, 1990; Li, 1994; Li, 1954-55; Lin, 1997; Liu, 1993; Ma, Wang, Sun, & Wang, 1991; Mei & Li, 1992; Siu, 1995; Wu, 1997; Xie & Tang, 1995; Yan, 1965; L. Zhao, 1991).)

Readers should note that mathematical knowledge was also transmitted in ancient China through channels other than the official school system. In the prefaces to some mathematical classics, references were made to learning from a master or even from a hermit or by self-education. Some historians of science argue that transmission through a religious network might play a considerable part (Needham, 1959; Volkov, 1996). Although the official system did produce tens of thousands of capable "mathocrats" who were employed as officials or imperial astronomers, almost all the eminent mathematicians who left their footprints in the history of mathematics seem to have been nurtured through other channels. An historian of mathematics once listed 50 Chinese mathematicians of fame who flourished between the 4th century B.C. and the end of the 19th century, with only two who can be labeled as educated in the official system (Guo, 1991).

Before closing this section, let us look at an unusual treatise which contains perhaps the first paper on mathematics education in China. The treatise was *cheng chu tong bian ben mo* (乘除通变本末, alpha and omega of variations on multiplication and division) written by the Song mathematician YANG Hui (杨辉) in 1274. The preface to the first chapter of the book is titled *xi suan gang mu* (习算纲目, A General Outline of Mathematical Studies). It offers a re-organized syllabus of the traditional curriculum accompanied by a time-table of a comprehensive study programme which takes only 260 days. This is comparable to a modern programme of about 1500 hours in secondary school mathematics. (Compare with the 7-year programme in the official system, which will be discussed in Section 3!) It is interesting and instructive to look at a few passages in this book, which explain quite well that rote learning is not to be equated with repetitive learning, and that doing a large number of exercises, is not incompatible with acquiring deep understanding. (The translated texts in the treatise of YANG Hui quoted below are taken from (Lam, 1977). See also (Zhou, 1990).)

> "In the *jia* (加, addition) method the number is increased, while in the *jian* (减, subtraction) method a certain number is taken away. Whenever there is addition there is subtraction. One who learns the 'subtraction' method should test the result by applying the 'addition' method to the answer of the problem. This will enable one to understand the method to its origin. Five days are sufficient for revision." (Book I, Chapter 1)

> "In knowing the *jiu gui* (九归, tables of division) one will need at least five to seven days to become familiar with the recitation of the forty-four sentences. However if one examines carefully the explanatory notes of the art on 'tables of division' in the *xiang jie suan fa* (详解算法, a detailed analysis of the methods of computation — a lost treatise by YANG Hui), one can then understand the inner meaning of the process and a single day will suffice for committing the tables and their applications to memory. Revise the subject on 'tables of division' for one day." (Book I, Chapter 1)

> "Learn a method a day and work on the subject for two months. It is essential to inquire into the origins of the applications of the methods so that they will not be forgotten for a long time." (Book I, Chapter 1)

> "The working of a problem is selected from various methods, and the method should suit the problem. In order that a method is to be clearly understood, it should be illustrated by an example. If one meets a problem, its method must be carefully chosen …. If numerical exercises are performed daily, this establishes a quicker insight into analyzing a problem and hence is beneficial to all." (Book I, Chapter 3)

"It is difficult to see the logic and method behind complicated problems. Simple problems are hereby given and elucidated. Once these are understood, problems, however difficult, will become clear." (Book II)

3 Official Curriculum in Mathematics in the Tang Dynasty

By the time mathematics was established as one of the disciplines of study in the official system in the Tang Dynasty, Chinese mathematics already enjoyed a scholarly tradition with a long history. In the middle of the 7th century the mathematician LI Chunfeng (李淳风) collated the *suan jing shi shu* (算经十书, Ten Mathematical Manuals) at an Imperial Order, and it was adopted as the official textbook in the School of Mathematics in 656. The 'Ten Mathematical Manuals' comprised ten classics compiled by different authors at different times, listed below roughly in chronological order: (1) *zhou bi suan jing* (周髀算经, The Arithmetical Classic of the Gnomon and the Circular Paths), 100 B.C., (2) 'Nine Chapters on the Mathematical Art' 100 B.C. to 100A.D., (3) *hai dao suan jing* (海岛算经, Sea Island Mathematical Manual), 3rd century, (4) *wu cao suan jing* (五曹算经, Mathematical Manual of the Five Government Departments), 6th century, (5) *sun zi suan jing* (孙子算经, Master Sun's Mathematical Manual), 4th century, (6) *xia hou yang suan jing* (夏侯阳算经, Xia Hou Yang's Mathematical Manual), 5th century, (7) *zhang qiu jian suan jing* (张丘建算经, Zhang Qiu Jian's Mathematical Manual), 5th century, (8) *wu jing suan shu* (五经算术, Arithmetic in the Five Classics), 6th century, (9) *qi gu suan jing* (缉古算经, Continuation of Ancient Mathematics), 7th century, (10) *zhui shu* (缀术, Art of Mending), 5th century. The original text of 'Art of Mending' was lost in about the 10th century. Its role in the 'Ten Mathematical Manuals' was subsequently replaced in the Song Dynasty by *shu shu ji yi* (数术记遗, Memoir on Some Traditions of the Mathematical Art), a book of doubtful 6th century authorship. (The original texts of (1) to (9) can be found in many references, for instance (Guo, 1993).) It is recorded in *xin tang shu* (新唐书, The New History of the Tang Dynasty) and *tang liu dian* (唐六典, The Six Codes of the Tang Dynasty) how these ten books were studied with specified duration. Students were divided into two programs, which for convenience will be denoted by A and B for short in this chapter. Students in Program A studied (1) to (8), *viz* ‘Master Sun’s Mathematical Manual’ and ‘Mathematical Manual of the Five Government Departments’ for 1 year, ‘Nine Chapters on the Mathematical Art’ and ‘Sea Island Mathematical Manual’ for 3 years, ‘Zhang Qiu Jian’s Mathematical Manual’ for 1 year, ‘Xia Huo Yang’s Mathematical Manual’ for 1 year, ‘The Arithmetical Classic of the Gnomon and the Circular Paths’ and ‘Arithmetic in the Five Classics’ for 1 year. Students in Program B studied (9) to (10), *viz* ‘Art of Mending’ for 4 years, and ‘Continuation of Ancient Mathematics’ for 3 years. In addition to these books, students in each of the two programs must also study two more manuals, *shu shu ji yi* (数术记遗, Memoir on Some Traditions of the Mathematical Art) and *san deng shu* (三等数, Three Hierarchies of Numbers). (The last manual was written not later than the mid 6th century but was lost by the Song Dynasty.) Regular examinations were held throughout the seven years of study, and at the end of each year an annual examination was held. Any student who failed thrice or who had spent nine years at the School of Mathematics would be discontinued. Judging from the age of admission at 14 to 19 years old, we know that a mathematics student would sit for the state examination at around 22. (For a more detailed discussion, see (Siu & Volkov, 1999).)

Although mathematics was included as one discipline in the official system, it received rather low regard. For instance, it was recorded in ‘The New History of the Tang Dynasty’ that 2 professors of mathematics and 1 teaching assistant of mathematics were appointed with 15 students admitted each year in each of Program A and Program B, while in the discipline of the classics 5 professors and 5 teaching assistants were appointed with 300 students admitted each year. If the number of the faculty and that of the student enrollment in itself do not bespeak the significance accorded to a discipline, the rank and salary of the faculty would. It was recorded in ‘The New History of the Tang Dynasty’ that a professor in mathematics was appointed as an official of the lowest rank (grade 30) while a teaching assistant was appointed with no official rank at all. But a professor in classics was appointed as an official of high rank (grade 11) and even a teaching assistant in classics was appointed as an official of only a slightly lower rank (grade 17)!

4 State Examination in the Tang Dynasty

The Chinese term for state examination is *keju* (科举), which means literally "subject-recommendation", i.e., recommendation of suitable candidates (for taking up official positions) through examinations in different subjects. Some historians date the beginning of the *keju* system to the Sui Dynasty when the emperor convened a state examination by decree. But some historians maintain that it started in 622 when the first Tang emperor decreed that any qualified candidate could sit for the state examination without having to be recommended by a provincial magistrate. As we will soon see, initially the *keju* system was a rather lively and efficient means for searching out and selecting capable persons to serve the country, based on their academic merit rather than their social background or hereditary aristocracy. However, in the long span of near to thirteen centuries of the operation of this system through different dynasties, it degenerated in later centuries into a kind of straitjacket of the mind which bred rote learning and a pedantic mindset. The *keju* system was abolished in 1905 by an imperial edict towards the end of the last imperial dynasty in China, the Qing Dynasty (Franke, 1968; Jin, 1990; Liu, 1996; Wu, 1997; Xie & Tang, 1995; Yang, Zhu & Zhang, 1992).

"One of China's most significant contributions to the world has been the creation of her system of civil service administration, and of the examinations which from 622 to 1905 served as the core of the system." (Kracke, 1947, p.103). Indeed, as early as in the beginning of the 17th century, the Jesuit Father, Matteo Ricci, reported with commendation in his journal "the progress the Chinese have made in literature and in the sciences, and of the nature of the academic degrees which they are accustomed to confer" (Ricci, 1615/1953). Voltaire (F.M. Arouet) made a similar observation in the mid 18th century, "The human mind certainly cannot imagine a government better than this one where everything is to be decided by the large tribunals, subordinated to each other, of which the members are received only after several severe examinations. Everything in China regulates itself by these tribunals." (Voltaire, 1756/1878, p.162). Dr. Sun Yat-Sen, founder of the Republic of China in 1911, said in The *Five-Power Constitution*, "At present, the civil service examination in the (Western) nations is copied largely from England. But when we trace the history further, we find that the civil service of England was copied from China. We have very good reason to believe that the Chinese examination system was the earliest and the most elaborate system in the world." (Teng, 1942-43, p.267). Dr. Sun even instituted the division of the government structure into five-powers, *viz* the Legislative Yuan, the Executive Yuan, the Judicial Yuan, the Examination Yuan and the Censorate.

Detailed official records of the *keju* system (in the Tang Dynasty) can be found in certain ancient chronicles, among which the main primary sources are:

jiu tang shu (旧唐书, Old History of the Tang Dynasty), 941–945;
xin tang shu (新唐书, New History of the Tang Dynasty), 1044–1058;
tang liu dian (唐六典, Six Codes of the Tang Dynasty), 738;
tong dian (通典, Complete Structure of Government), 770–801;
tang hui yao (唐会要, Collection of Important Documents of the Tang Dynasty), 961.

One informative secondary source which contains the main excerpts of relevant interest in the chronicles listed above plus a lot more of interesting information and anecdotes is the *deng ke ji kao* (登科记考, Journal on the Examinations in the Tang Dynasty) compiled by the Qing Scholar XU Song (徐松) in 1838. (Most of the anecdotes given in this section can be found in this book (Xu, 1838/1984).) In Western literature one of the earliest works on the state examination system in the Tang Dynasty is that of the famed French sinologist Edouard Biot, who did not seem to have a high regard for the official curriculum. He claimed that "the name of School of Mathematics is too high-sounding for the studies in this elementary establishment" and that the adopted textbooks were "collections of problems which are for the most part elementary and whose solutions are given without proofs" (Biot, 1847/1969, pp.257, 262). The first comprehensive account in a Western language of the state examination system in the Tang Dynasty, together with a reliable translation of the relevant documents, was provided by Robert des Rotours in 1932 (des Rotours, 1932).

In 'The New History of the Tang Dynasty' a section on recruitment by examinations records that there were two kinds of state examinations: (1) regular examinations held annually in the first or second lunar month for graduates of colleges and universities or for provincial candidates, (2) special examinations held by imperial decree. The second kind depended on the need at the time or on the whim of the emperor, so it covered a wide range of expertise, but could also sound rather strange. In official records one can find about a hundred of such special examinations. Just to cite a few, there were: examination on "vast erudition and great composition", examination on "deep knowledge of the ancient books and great talents in the art of teaching", examination on "having military plans with foresight and well qualified as a general", examination on "wisdom and good nature, rectitude and righteousness, and speaking honestly and remonstrating insistently", examination on "remarkable understanding of the art of government and suitability for administering people". A most amusing item is examination on "leading an hermetic life at Qiuyuan, not seeking fame", since logically speaking one should be awarded a degree in that if and only if one should not be! (In fact, it was recorded in 'Journal on the Examinations in the Tang Dynasty' that somebody was awarded the degree *in absentia* in 794 as he refused to receive it!) For the first kind there were initially seven subjects: examination on perfect talent, examination on classics, examination on distinguished man of letters, examination on accomplished man of letters, examination on law, examination on calligraphy and examination on mathematics. Examination on perfect talent was soon abolished, while examination on accomplished man of letters became in time the main focus enjoying the highest prestige. It was recorded in 'Complete Structure of Government' that by 752, of a thousand candidates who sat for the annual examination on accomplished man of letters only one or two were awarded the degree, while for instance, successful candidates for the examination on classics numbered in the tens. A source of the time said that one who passed the examination on accomplished man of letters at fifty (perhaps after many repeated attempts) was still regarded as outstanding, while one who passed the examination on classics at thirty was considered too old already! No similar data or remark is found for examination on mathematics, which again serves to indicate that mathematics was accorded a lower prestige among the various subjects, only on a par with calligraphy. This becomes even more apparent when we look at the number of students enrolled at the state university. Tang institutions of higher education were divided into hierarchies. The highest institution was the School for the Sons of the State which accepted only sons of noblemen or officials from a certain rank upward. Next came the National University which accepted a similar crop with the official rank somewhat lowered. Then came the School of Four Gates which accepted, besides sons of officials, also a small number of sons of ordinary citizens. The three Schools of Law, Calligraphy and Mathematics accepted sons of officials of low rank and of ordinary citizens. In the early Tang Dynasty, according to the 'New History of the Tang Dynasty', there were 300 students in the School for the Sons of the State, 500 students in the National University, 1300 students in the School of Four Gates, 50 students in the Law School, 30 students in the Calligraphy School and 30 students in the Mathematics School. At one time, throughout the whole empire, including the provincial colleges, there were 8000 students pursuing higher education, with foreign students coming from nearby countries as well. The whole edifice of state higher education was very well-established in the Tang Dynasty.

The culminating apex of this edifice, the annual state examination, was a grueling experience for many. Some authors in the Tang Dynasty wrote about how candidates stood in a long queue, carrying their own stationery, supply of food and water, candles and charcoal (for preparing meals and for getting warmth), waiting to be admitted to their cells, only to be searched and shouted at by guards who were stationed by the thorny hedge (an ancient analogue of barbed-wire fence) which encompassed the examination venue. Candidates were clad in flimsy clothes and shivered in the freezing weather, for they were not allowed thick clothing to prevent concealment of manuscripts. Throughout the long hours they worked on their examination scripts, the candidates were confined to their cells, in which they would prepare their own meals and take care of their own personal hygiene. In the case of failure in the examination, which was not uncommon, this grueling experience would have to be repeated in another year, and perhaps in yet another year, …. WEI Chengyi (韦承贻), who was awarded the degree of accomplished man of letters in 867, once sneaked into the office of the Ministry of Rites called Nangong, which was in charge of examination affairs, and composed a poem on the wall: “Like a thousand white lotus petals, /The candles lit up the hall, /Which was filled with the peaceful rhymes/Of the Ya and the Song./As the flame of the third candle/Flickered towards its end, /One realized it meant failure/To complete the scene of Nangong.” (白莲千朵照廊明。一片升平雅颂声。才唱第三条烛尽。南宫风景画难成。) This poem, with its trace of resignation, depicted vividly those assiduous candidates racing against time with their examination scripts by the light of the three candles allowed them to last through the night.

Modern examinations are definitely much less grueling than that. However, it would be unfair to his ancestors in the Tang Dynasty if the author fails to point out that even over a thousand years ago some good modern examination procedures were already in place. In 759 the Chief Examiner LI Kui (李揆) said, “The empire selects its officials for their talent. The requisite classics are displayed here. Candidates are welcome to consult them at will.” This was perhaps the earliest open-book examination! In 742 the Chief Examiner WEI Zhi (韦陟) said, “The performance of a candidate in one single examination may not reflect his true potential, hence his previous essays should also be consulted.” This was perhaps the earliest instance of assessment by project work and portfolio! A famous example is the work presented by BAI Juyi (白居易) to the Chief Examiner GU Kuang (顾况). BAI Juyi was awarded the degree of the accomplished man of letters in 800 at a rare early age of 27, and is remembered today as a renowned poet of the 9th century. The “project work” he presented is handed down as one of his very well-known and oft-quoted poems which begins (translated text taken from Xu & Yuan, 2000): “Grass on the plain spreads higher and higher; /Year after year it fades and grows./ It can’t be burned up by wild fire, / But revives when the spring wind blows./ …” (离离原上草。一岁一枯荣。野火烧不尽。春风吹又生。)

5 State Examination in Mathematics in the Tang Dynasty

In the state examination for either Program A or B, in mathematics, the candidate was examined on two types of question. The first type was described in the ‘New History of the Tang Dynasty’ as: “[The candidates should] write [a composition on] the general meanings, taking as the basic/original task a ‘problem and answer’. [They should] elucidate the numbers/ computations, [and] construct an algorithm. [They should] elucidate the structure/principle of the algorithm in detail.” (録大义本条为问答。明数造术。详明术理。) For Program B there was added the remark, “If there is no commentary, [the candidates should] make the numbers/computations correspond [to the right ones?] in constructing the algorithm.” (无注者合数造术。不失义理。) (For an attempt to explain the latter remark, see (Siu & Volkov, 1999).) We will say more about this type of question in Section 6. The second type of question was testing on quotations known as *tie du* (帖读, strip reading). Candidates were shown a line taken from either *shu shu ji yi* or *san deng shu*, with three characters covered up. Candidates had to answer what those three characters were. In to-day’s terminology, this type of questions is called “fill in the blank”. It is interesting to note that the ‘Memoir on Some Traditions of the Mathematical Art’ is a short text with only 934 characters, which could be committed to memory with reasonable ease (not to mention that a candidate had seven years to do it!). There may well be other reasons for singling out this book for the purpose of testing on quotations, but that would be the subject of another paper. (See (Volkov, 1994) for an interesting discussion on the content of ‘Memoir on Some Traditions of the Mathematical Art’) The book ‘Three Hierarchies of Numbers’ was lost by the Song Dynasty (960 – 1279). We can only surmise that it might be a text similar to the ‘Memoir on Some Traditions of the Mathematical Art’ in this respect.

By the way, there was a reason for instituting the practice of testing on quotations. The practice was proposed by the Chief Examiner LIU

Sili (刘思立) in 681 (in all subjects) to rectify the deficiency of a prevalent habit of candidates who only studied "model answers" to past questions instead of studying the original classics. Testing on quotations forced candidates to read (at least some) original classics. However, examination being what it is, it is prone to abuse. The setting of questions on quotations got more and more difficult and unreasonable, testing candidates on obscure phrases, sometimes even setting up traps to confound the candidate intentionally. In response, candidates collected such obscurities and memorized them for the sole purpose of passing those unreasonable tests! The laudable purpose of encouraging candidates to read the original classics was totally defeated. In 728 it was decreed that quotations should be set within reasonable bounds. There is a good lesson to be learnt here about making use of examination to direct the curriculum.

6 A "Re-constructed" Examination Question

Since no trace of any examination question is extant, but there is a reference to tasks on elucidation and construction in the state examination in mathematics, we will attempt to "re-construct" some examination questions to lend evidence in support of the thesis that the curriculum in mathematics in the Tang Dynasty was not so elementary nor was it learnt by rote. It is hard to imagine that a group of selected young men spent seven of their golden years in simply memorizing the mathematical classics one by one without understanding just to regurgitate the answers in the state examination at the end! If readers are of the opinion that imagination should have no place in historical study, the author will insert an (apologetic?) self-defense at this point by referring to a wider (but somewhat controversial) view of studying history as propounded by the British philosopher-historian Collingwood (Collingwood,1946, p.202), "History is thus the self-knowledge of the living mind. … For history is not contained in books or documents; it lives only, as a present interest and pursuit, in the mind of the historian when he criticizes and interprets those documents, and by so doing relives for himself the states of mind into which he inquires."

Collingwood echoed the view held by the Italian philosopher Croce who said in (Croce, 1919/1920, p.19), "History is living chronicle, chronicle is dead history; history is contemporary history, chronicle is past history; history is principally an act of thought, chronicle an act of will. Every history becomes chronicle when it is no longer thought, but only recorded in abstract words, which were once upon a time concrete and expressive."

Before giving examples, it is helpful to look at a typical textbook and see how the author did mathematics. Which better choice can one pick than the prime textbook 'Nine Chapters on the Mathematical Art' (with English translation in Shen, Crossley & Lun, 1999)? With the commentaries by the 3rd century mathematician LIU Hui added, this provides more "circumstantial evidence" for our thesis.

In Chapter 5 of the 'Nine Chapters on the Mathematical Art' some formulae for the volume of various solids are given. In particular, Problem 17 is about that of a tunnel at the entrance of a tomb (*xianchu* 羡除). Mathematically speaking, a *xianchu* is a solid bounded by three trapeziums and two triangles on the two sides. The three trapeziums have opposite parallel sides of length a, b; a, c and b, c, the depth is h and the trapezium on top has length l (see Figure 1).

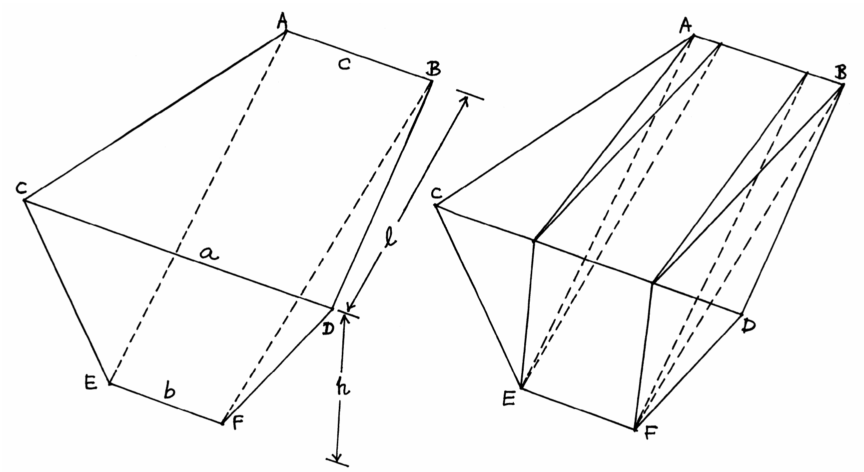

Figure 1.

The formula for the volume of the *xianchu* is given in the text as $V = \frac{1}{6}(a+b+c)hl$. (In the text, numerical data are given in place of *a*, *b*, *c*, but the numerical data are actually generic rather than special.) LIU Hui explains in his commentaries how the volume is calculated. He dissects the *xianchu* into smaller pieces, each of some standard shape such as a triangular prism (*qiandu*, 堑堵), a tetrahedron of a particular type (*bienao*, 鳖臑), or a pyramid with a square base (*yangma*, 阳马). If you try to do that by yourself, you will find out that the way of dissection is different for different relations between *a*, *b*, *c*. For instance, if $a > c > b$, then you obtain two tetrahedron of a particular type each of volume $\frac{1}{12}(a-b)hl$, two tetrahedron of a particular type each of volume $\frac{1}{12}(c-b)hl$ and one triangle prism of volume $\frac{1}{2}bhl$ (see Figure 1.) They add up to $\frac{1}{6}(a+b+c)hl$. But if $a > b > c$, then you obtain two tetrahedron of a particular type each of volume $\frac{1}{12}(a-b)hl$, two pyramid with a square base each of volume $\frac{1}{6}(b-c)hl$ and one triangular prism of volume $\frac{1}{2}chl$. They also add up to $\frac{1}{6}(a+b+c)hl$. In fact, LIU Hui in his commentaries treats all eight different cases except the one case $b > a = c$. The calculation is different for different ways of dissection, but the basic underlying idea is the same. Probably candidates in the examination would be asked to carry out a similar explanation for other formulae on area and volume, possibly with given numerical data. Once the basic idea is understood, such a demand for elucidation is reasonable.

In the same chapter, Problem 10 is about the volume of a pavilion (*fangting*, 方亭) with square base (see Figure 2).

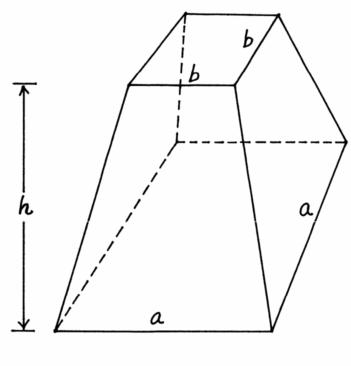

Figure 2.

Mathematically speaking, a fangting is a truncated pyramid with square base. If *a*, *b* are the sides of the bottom and top squares respectively and *h* is the height, then the volume is given in the text as $V = \frac{1}{3}(a^2 + b^2 + ab)h$. Again, LIU Hui in his commentaries explains how to obtain the formula by an ingenious method of assembling blocks of standard shape (called by him *qi*, 棋). There are three kinds of *qi*: cube of side a with volume a^3 (*lifang*, 立方, LF); pyramid of square base of side *a* and one vertical side of length a perpendicular to the base, with volume $\frac{1}{3}a^3$ (*yangma*, YM); triangular prism with isosceles right triangle of side *a* as base and height *a*, with volume $\frac{1}{2}a^3$ (*qiandu*, QD). He observes that the truncated pyramid is made up of one LF, four YM and four QD. (Careful readers will notice that here we require $h = b$, so that we are talking about blocks of a standard shape.) He then observes that one LF makes up a cube of volume b^2h; one LF and four QD make up a rectangular block of volume *abh*; one LF, eight QD and twelve YM make up a rectangular block of volume a^2h. (Careful readers will notice that here we require $h = b$ and $a = 3b$ so that each corner piece is a cube formed from three YM.) In problem 15, LIU Hui further explains how to obtain the more general formula of the volume of a pyramid of rectangular base with an arbitrary height by an infinitesimal argument (Wagner, 1979). Altogether, three LF, twelve QD and twelve YM make up a volume $b^2h + abh + a^2h$. Hence the volume of the truncated pyramid is $\frac{1}{3}(a^2 + b^2 + ab)h$ (see Figure 3).

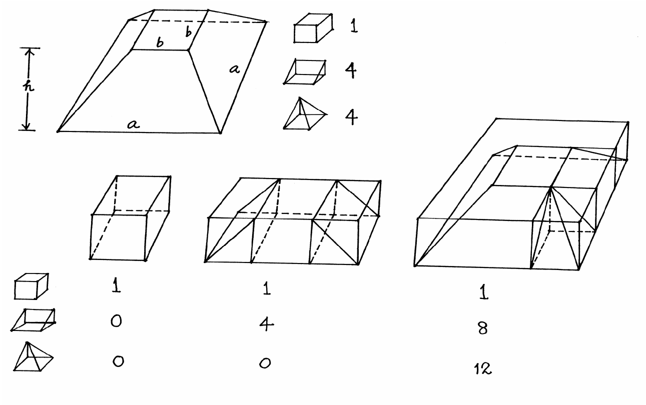

Figure 3.

LIU Hui gives an alternative formula $V = \frac{1}{3}(a-b)^2 h + abh$ by another way of dissection (see Figure 4).

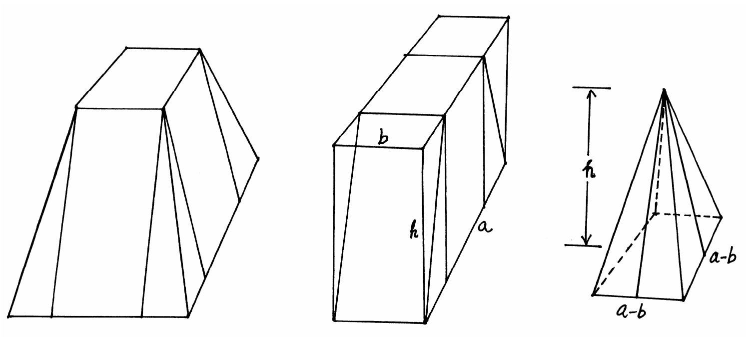

Figure 4.

In the second explanation, there is no need to assume $h = b$, $a = 3b$. But it works only when the bottom and top pieces are squares.

Here is a fictitious examination question: Compute the volume of an "oblong pavilion" of height h with bottom and top being rectangles of sides a_1, a_2 and b_1, b_2 respectively ($a_1 \neq a_2$, $b_1 \neq b_2$). If one understands the argument by LIU Hui, one can easily modify either method to arrive at the answer, which is left as an exercise for the readers. (Readers may also wish to solve the problem in a way commonly known to school pupils of today, *viz* by making use of similar triangles.) The answer turns out to be $V = \frac{1}{3}\left[a_1a_2 + b_1b_2 + \frac{1}{2}(a_1b_2 + a_2b_1)\right]h.$ If one merely memorizes the formula given in the textbook by heart, it is not easy to hit upon the correct formula. This is probably what is meant by "constructing a (new) algorithm". Again, such a demand is reasonable, especially when these candidates might well be facing in their subsequent career problems which are variations (e.g. with parameters changed) of the problems they learnt in the textbooks.

7 Is the State Examination Really so Damaging to Learning?

The *keju* enjoys the strange ambivalence of being described as a rich cultural heritage as well as a scandalous historical burden of the Chinese, with its strength and shortcoming the subject of controversy to this date (Jin 1990; Liu, 1996). We will not enter into another long debate here. To counteract the traditional view that the ancient Chinese studied mathematics only by rote memorization and industrious drilling, we will, however, "magnify" the more positive part of the system, which unfortunately was outweighed by its negative part, especially in later evolution in the Ming and Qing dynasties.

It is strange that a popular view is to equate Confucian learning with rote learning and with submissive learning despite what the Masters themselves had said. (The following translated texts are taken from (Legge, 1893/1960).) In the 'Confucian Analects' (论语, 5th century B.C.) we find, "Learning without thought is labor lost; thought without learning is perilous." In the 'Doctrine of the Mean' (中庸, 6th-5th century B.C.) we find, "He who attains to sincerity, is he who chooses what is good, and firmly holds it fast. To this attainment there are requisite the extensive study of what is good, accurate inquiry about it, careful reflection on it, the clear discrimination of it, and the earnest practice of it." In the books by the leading neo-Confucian scholar ZHU Xi (朱熹) (1130 – 1200) we find (all translated texts from the books of

ZHU Xi quoted below are taken from (Gardner, 1990)), "In reading, if you have no doubts, encourage them. And if you do have doubts, get rid of them. Only when you've reached this point have you made progress." (Book 11, p.151). Would one call this rote-learning? submissive learning? By reading more extensively in the books by ZHU Xi, we can perhaps understand better what appears to Western observers as rote-learning actually consists of. ZHU Xi said, "Generally speaking, in reading, we must first become intimately familiar with the text so that its words seem to come from our own mouths. We should then continue to reflect on it so that its ideas seem to come from our own minds. Only then can there be real understanding. Still, once our intimate reading of it and careful reflection on it have led to a clear understanding of it, we must continue to question. Then there might be additional progress. If we cease questioning, in the end there'll be no additional progress." (Book 10, p.135). He also elaborated further, "Learning is reciting. If we recite it then think it over, think it over then recite it, naturally it'll become meaningful to us. If we recite it but don't think it over, we still won't appreciate its meaning. If we think it over but don't recite it, even though we might understand it, our understanding will be precarious. … Should we recite it to the point of intimate familiarity, and moreover think about it in detail, naturally our mind and principle will become one and never shall we forget what we have read." (Book 10, p.138). This is an unmistakable differentiation between repetitive learning and rote learning. Contemporary researchers explain the Asian Learner Paradox based on this differentiation (Biggs 1996; Marton, Dall' Alba, & Tse, 1996).

On the other hand, modern day education in the Western world which arose in the 19th century along with the Industrial Revolution started by emphasizing the 3Rs — reading, writing and arithmetic. In a code issued by Robert Lowe of the Education Department of England in 1862, specific standards for each R were explicitly stated (e.g. Standard I in Reading: Narrative monosyllables; Standard II in Writing: Copy in manuscript character a line of print; Standard IV in Arithmetic: A sum in compound rules [money]) (Curtis, 1967, Chapter VII). The emphasis on mechanical rote learning is captured vividly in the opening sentences (which were intended as a satirical exaggeration) of the 1854 novel *Hard*

Times by Charles Dickens (as words uttered by Mr. Gradgrind of Coketown)

> "Now, what I want is, Facts. Teach these boys and girls nothing but Facts. Facts alone are wanted in life. … This is the principle on which I bring up my own children, and this is the principle on which I bring up these children. Stick to Facts, sir!" (Dickens, 1854/1995, p.9)

About the preparation for state examinations, ZHU Xi has also left us with the following passages (all translated texts from the books of ZHU Xi quoted below are taken from (Gardner, 1990)):

> "Scholars must first distinguish between the examinations and studying, which is less important, which is more important. If 70 percent of their determination is given to study and 30 percent to the examinations, that'll be fine. But if 70 percent is given to the examinations and 30 percent to study, they're sure to be overcome by the 70 percent." (Book 13, p.191)

> "Preparing for the examinations doesn't harm one's studying. Previous generations, when did they ever refrain from taking the examinations? It's simply because people today don't settle their minds that harm is done. As soon as their minds become fixed on success or failure in the examinations, their understanding of the words they read is all wrong." (Book 13, p.194)

> "He was once discussing the examinations and said: It isn't that the examinations are a trouble to men, it's that men become troubled by the examinations. A scholar of superior understanding reads the texts of the sages and worthies and on the basis of his understanding of them writes the essays required in the examinations. He places aside considerations of success and failure, gain and loss, so even if he were to compete in the examination every day he wouldn't be troubled by them. If Confucius were born again in today's world, he wouldn't avoid

> competing in the examinations, and yet they wouldn't trouble him in the least." (Book 13, p.194).

Over 800 years ago the Chinese sages already knew that the main shortcoming of examination does not come from the examination itself but from the high stakes it brings with it!

Granted that an examination is not to be passed through rote learning, what good will an examination bring? Let us first compare the ancient Chinese examination format with a modern theory on assessment by Bloom (Bloom, 1956). The modern viewpoint includes both the formative and the summative aspects of assessment, while the ancient Chinese examination focused only on the latter function for selection purposes. The six major classes of taxonomy of Bloom can be matched up with the four different types of question in the ancient Chinese examination, *viz* (i) testing on quotations is about knowledge, (ii) short questions are about comprehension and application, (iii) long questions (on contemporary affairs) are about analysis and synthesis, (iv) composition and poems are about evaluation (Liu, 1996, p.240).

With these varied objectives, an examination can have a beneficial influence on both the student and the teacher, even as a summative process. For the student it is good for consolidation of knowledge, enhancement of comprehension, planning of schedule of study, judgment on what is important to learn, development of learning strategies and motivation and self-perception of competence. For the teacher, besides what has been said above, it is good for monitoring the progress of the class, as a gauge of the receptivity and assimilation of the class and evaluation of the teaching. In this sense, "examination-oriented education" and "quality education" need not be a dichotomy. Crooks says, "As educators we must ensure that we give appropriate emphasis in our evaluations to the skills, knowledge, and attitudes that we perceive to be most important." (Crooks, 1988, p.470). Viewed in the summative aspect, an examination is a necessary evil. But viewed in the formative aspect, an examination can be a useful part of the learning process. Moreover, it is a false dichotomy to differentiate strictly summative assessment and formative assessment. The important thing to keep in mind is not to let the assessment tail wag the educational dog! (Tang &

Biggs, 1996, p.159). The demise of the examination system in Imperial China, even with its initial good intention and with its long life span of 1287 years, is a lesson to be learnt from.

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