

THE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

YSCN0002: Mathematics: A Cultural Heritage

Lecturer: Professor SIU Man Keung (RR 4-18)

Demonstrator: nil

Welcome to this course!

In the words of Hermann Weyl (1885-1955): “[But] mathematics sets the standard of objective truth for all intellectual endeavours; science and technology bear witness to its practical usefulness. Besides language and music, it is one of the primary manifestations of the free creative power of the human mind, and it is the universal organ for world-understanding through theoretical construction. Mathematics must therefore remain an essential element of the knowledge and abilities which we have to teach, of the culture we have to transmit, to the next generation.” This course attempts to elaborate this exhortation through examples gathered from the long history of mathematics, around our daily lives, in other areas of human endeavours and in Nature. Rather than transmitting a body of technical knowledge in mathematics the emphasis is placed on appreciating, contemplating and discussing about the beauty, the utility and the “Way” of mathematics. The demand on technical preparation in mathematics is minimal, say up to the level of general mathematics in school, but the student is expected to possess some degree of intellectual curiosity and willingness to participate in the reasoning process. I shall try to make it a pleasurable and enlightened learning experience, but **YOU** yourself have an important part to play in making this possible.

The class will meet for two consecutive class sessions with a break in between every Wednesday afternoon. There will be about 12 to 14 lectures and 8 to 10 tutorials. With a large class, by a tutorial I do not mean small group discussion, much as I would wish it were. Rather, it will be run as a guided discussion session *en masse* with the provision of a tutorial sheet in advance on web. [Please make it a habit to consult the webpage of YSCN0002 weekly. Go to <<www.hku.hk/math/>> and click the item on “Undergraduate Programmes and Courses”. To save paper (or not to violate copyright law) all course material will be put on web or deposited (one hard copy) in the Reserved Section of the University Library. Search for the course title on the library catalogue.] Try to participate in the tutorial by going through the tutorial sheet before coming to class. From time to time towards the end of the class session you may be asked to write down a few sentences on your views, comments or questions about what we discuss in that class session. There will also be a few homework assignments spread out in the semester. These are to be handed in on a group basis rather than on an individual basis. [Frankly I cannot manage to go through a large number of scripts each time! I would therefore request your cooperation in forming yourselves into groups with about 3 to 4 persons to one group. Each group will hand in one piece of work for each homework assignment. In the case where a member of the group dissents considerably from what is handed in as a group, that member is welcome to make such a remark and express his or her own view.

If you really want to work in a small group with two or even just one member, you can do so.] Hand in homework by the due date, and **LATE homework will NOT be graded.** You can deposit your homework in the Assignment Box labelled YSCN0002 opposite the lift of 4/F of Runrun Shaw Building. If you hand in your homework electronically, you can use the email address <<mathsiu@hkucc.hku.hk>>. With these many opportunities as described above to observe the effort you put into the course, I will give the coursework assessment grade which counts for 50%, while the remaining 50% will be assessed on an essay about **one** book of your choice. This essay is to be done on an individual basis and not on a group basis. The main text of your essay (not counting figures and bibliography) should be of length between 2000 to 3000 words, and it is to be handed in **on or before May 4, 2009**. You should have ample time to select and to read your book. In due course I will announce some guidelines on the selection of books along with some suggestions.

There will be no final written examination. Lectures and tutorials are therefore not to prepare you for a final written examination but to nurture in you a sense of mathematics so that you will feel more at home in doing your homework assignments and to choose your book, to read it and to write on it. The main difficulty in teaching this course lies with the wide range of interest and expectation among individual students in the class because of their varied background and motivation. Some may like to see more technical details in mathematics, while some may get scared by that. Some may like to see the more philosophical aspect of mathematics, while some may like to see actual applications in their own fields of interest or to see daily occurrence of mathematics. Some may like to see mathematical proofs of certain theorems, while some may try to avoid that. Some may like to learn more mathematics, while some may only want to get a general idea of what mathematics is about. Some may just wish to have a general appreciation of mathematics. I try to have some of everything, but obviously that cannot satisfy everybody at all times. I can just hope that in general not too many students feel either bored or alienated most of the time, and that occasionally a topic will catch the fancy of some students at some time, perhaps different students at different times. If occasionally the discussion of a topic does resonate with you, then I regard the course as having achieved its purpose. If I fail to do that so that most of the class feel bored or alienated most of the time, then the course is a failure. You can help me to prevent that happen by talking with me when you have questions.

I am no authority. I am not teaching you any specific topic nor instructing you on any special technique or any specific theorem. (You acquire that kind of knowledge in a mathematics course of a more technical nature. You do not expect to acquire it from a broadening course. If you do pick up some mathematics on the way, then it is a bonus.) I try to offer you many many examples and instances to stimulate your own thinking and to get you immersed in an environ of mathematics. These examples and instances range from your daily experience to contact with other areas of intellectual pursuit, to a deeper and more philosophical thinking in the realm of mathematics, as the course unfolds in the next three months.

Don't worry if you cannot copy down in class everything shown on a slide, which is after all a teaching aid and is not meant to be copied down in totality. In many cases the slides are used to enliven the lecture with pictures and the like. I will place on web a summary of what we discuss in each class session before we meet for the next class session. What you should do is to read from books. Try to make good use of the library.

There is no dearth of books on mathematics for a general reader. The following list covers only a small portion, and I choose only those that are within reach of one whose education in mathematics is up to school level. (If you go beyond that, so much the better.) Books which go into more in-depth discussion will be mentioned in class for those who wish to probe further. To save you the time in locating these items, I append their respective call numbers (in the HKU library) in brackets at the end.

- E.T. Bell, *Mathematics: Queen and Servant of Science*, McGraw-Hill, 1951; republished by Math. Asso. Amer. in 1987. (510.9 B43m)
- D.M. Campbell, J.C. Higgins (eds.), *Mathematics: People, Problems, Results*, Vol. 1-3, Wadsworth International, 1984. (510 C18)
- P.J. Davis, *Mathematics and Common Sense: A Case of Creative Tension*, A.K. Peters, 2006 (511.3 D2)
- P.J. Davis, R. Hersh, *The Mathematical Experience*, Birkhäuser, 1980; Study Edition (with E.A. Marchisotto), 1995. (510.1 D26)
- K.J. Devlin, *Life By the Numbers*, Wiley, 1998. (510 D4971)
- H. Eves [and C.V. Newsom], [*An Introduction to the*] *Foundations And Fundamental Concepts of Mathematics*, 3rd edition, PWS-Kent Publishing, 1990; originally published by Holt, Rinehart & Winston in 1958, revised in 1965. (510.1 E9, 510.1 E93i)
- T. Gowers, *Mathematics: A Very Short Introduction*, Oxford University Press, 2002. (510 G7)
- L.T. Hogben, *Mathematics For the Million*, Allen & Unwin, 1936; 3rd ed., 1951. (510 H715m, [X] 510 H715m)
- D. Huff, *How To Lie With Statistics*, Norton, 1954. (311.2 H88)
- J.P. King, *The Art of Mathematics*, Plenum Press, 1992. (510 K53)
- M. Kline, *Mathematics in Western Culture*, Oxford University Press, 1953. (510.9 K65m)
- L. R. Lieber, *The Education of T.C. Mits: What Modern Mathematics Means to You*, Galois Institute Press, 1942; revised edition, W.W. Norton, 1944; republished by Paul Dry Books, 2007.
- J.R. Newman (ed.), *The World of Mathematics*, Vol. 1-4, Simon & Schuster, 1956. (510.2 N55w)
- J.A. Paulos, *Innumeracy: Mathematical Illiteracy and Its Consequences*, Vintage Books, 1990; originally published in 1988. (510 P33i)
- G. Pólya, *How To Solve It*, 2nd ed. Princeton University Press, 1957; reprinted by Penguin, 1990; originally published in 1954. (510.7 P78h, [E] 510.7 P78h)
- G. Shaffner, *Arithmetic of Life*, Ballantine Books, 1999. (646.7 S5.2)
- S.K. Stein, *Strength in Numbers: Discovering the Joy and Power of Mathematics in Everyday Life*, Wiley, 1996. (510 S819V)
- R. Taschner, *Numbers at Work: A Cultural Perspective*, A.K. Peters, 2007 (translation of the German original *Die Zahlen Gigantische Schatten* published in 2005). (510.9 T197 n97)
- 羅浩源, <<生活的數學>>, 香港教育圖書公司, 1997. ([中] 410.7 60)
- 蕭文強, <<為什麼要學習數學?—數學發展史給我們的啟發>>, 修訂本, 九章出版社, 1995; 學生時代出版社, 1978; 香港新一代文化協會重版, 1992. ([E] 410.7 443-1, [中] 410.7 443)

I will also put down three general accounts on history of mathematics.

- H. Eves, *An Introduction To the History of Mathematics*, 6th edition, Saunders, 1990; originally published by Holt, Rinehart & Winston in 1953. (510.9 E93, [E] 510.9 E93)
- I. Grattan-Guinness, *The Fontana History of the Mathematical Sciences: The Rainbow of Mathematics*, Fontana Press, 1997. (510.9 G7f)
- 劉鈍, <<大哉言數>>, 遼寧教育出版社, 1993. ([中] 410.92 72)

There are many webpages on mathematical topics or mathematical news for the general public. Being brought up in this IT age, you probably are much more proficient than I in such search. I welcome any information and hint from you. But when surfing on the internet, please remember not to equate information with knowledge, or knowledge with wisdom.

Course content of YSCN0002 (Mathematics : A Cultural Heritage)

(It is difficult to lay down once and for all a fixed course schedule. Selection of topics depends on the interest of the class, involvement of the class, number of available class sessions, and hence topics may vary from year to year. Along with the variation assignments and tutorial worksheets, which form an integral part of the course, may also vary from year to year. What is given below is therefore an outline of the course content rather than a day-by-day topic-by-topic account.)

*** Introduction**

- mathematics and culture (in both East and West)
- the utility, the beauty and the “Way” of mathematics

*** Mathematics around us**

- clippings from newspapers and magazines with a mathematical connection, accompanied by discussion on the related mathematics

*** Knowledge in mathematics, sense of mathematics and the Anschauung of mathematics**

- illustration of the three aspects through examples

*** Beauty of Mathematics**

- external and extrinsic aspect (in Nature, in fine arts, music, architecture and in applications)
- internal and intrinsic aspect (within mathematics)
- illustration of the two aspects through examples

*** Mathematics and literature**

- examples of allusion to mathematics in literature (novels and poems) both on the negative side or on the positive side, some with a philosophical touch, some with a pedagogical touch, some with a satirical touch, some with a witty touch, accompanied by discussion of the examples in their literary or

historical contexts

* **Mathematical thinking**

- logical thinking
- axiomatic approach
- mathematical creation
- mathematical proof

* **Influence of mathematics on other human endeavour (not so much as a technical tool but as a way of thinking) and vice versa**

- mathematics and social sciences
- mathematics and philosophy
- the story of the discovery of non-euclidean geometry and its philosophical implication

Guidelines on your essay on a book of your choice (to be handed in by **May 4, 2009**)

1. As explained in the **Course Information**, the main text of your essay (excluding figures and bibliography) should be of length between 2000 to 3000 words. Each student submits his or her own essay on an **individual basis**.
2. The book you select to write on should **not** be a book of technical content on a certain subject in mathematics *per se*. For instance, do not select a book with a title like *Introduction to Number Theory* or *A First Course in Calculus*.

A possible selection includes a popular account on mathematical topics, an exposition on the nature or on the philosophical aspect of mathematics, a novel or a play with a mathematical flavour, biographies of mathematicians, There is no dearth of this genre of books. The level may vary with the background of mathematical knowledge and mathematical maturity of the readership the author has in mind. In this course I have a fairly general audience in mind. But if you wish to choose a book that demands more on your mathematical knowledge, by all means do it.

The list below is **NOT** to be regarded as a recommended booklist from which you select one to write on. Rather, treat it as a list of examples. By browsing over some of them you will get a rough idea what I mean, and you can select any suitable book which is not necessarily on this list, as long as it attracts you. Almost all books in the list are available in the University Library, but you need not restrict your own choice to the holding in the University Library. You can find some such books given in the **Course Information** or mentioned in class (see the **Summaries**). In addition let me list a few novels or collection of short stories or plays:

- E.A. Abbott, *Flatland: A Romance of Many Dimensions* (2nd revised edition, 1884),
- D. Auburn, *Proof* (2001),

- C. Djerassi, *The Bourbaki Gambit* (1994),
- A. Doxiadis, *Uncle Petros and Goldbach's Conjecture* (2000),
- C. Fadiman, *Fantasia Mathematica*. (1958),
- D. Guedj, *The Parrot's Theorem* (2000),
- A.P. Lightman, *Good Benito* (1994),
- Y. Ogawa (小川洋子), *The Professor's Beloved Formula* (orig. in Japanese 2003, trans. in Chinese, French and English; English title *The Gift of Numbers*).

Let me also list several biographies of mathematicians of the modern era. (There are some books on history of mathematics mentioned in the **Course Information**. Those are about the work of mathematicians of a more distant past.)

- S. Batterson, *Stephen Smale: The Mathematician Who Broke the Dimension Barrier* (2000)
- P. Hoffman, *The Man Who Loved Only Numbers: The Story of Paul Erdős and the Search for Mathematical Truth* (1998)
- R. Kanigel, *The Man Who Knew Infinity: A Life of the Genius Ramanujan* (1991)
- S. Nasar, *A Beautiful Mind* (1998)
- C. Reid, *Hilbert* (1970)
- B. Schechter, *My Brain is Open: The Mathematical Journeys of Paul Erdős* (1998)
- H. Wang, *Reflections on Kurt Gödel* (1987)
- Y. Wang, *Hua Loo-Kang* (1994; translated into English by P. Shiu, 1999)

For popular accounts on mathematics, besides those titles listed in the **Course Information**, I can recommend a few classics, but they may demand of the reader mathematical knowledge at some level:

- R. Courant and H. Robbins, *What is Mathematics?* (1948; 2nd revised edition by I. Stewart, 1996)
- D.R Hofstadter, *Gödel, Escher, Bach: An Eternal Golden Braid* (1979)
- T.W. Körner, *The Pleasure of Counting* (1996)
- R. Osserman, *Poetry of the Universe: A Mathematical Exploration of the Cosmos* (1995)
- I. Stewart, *The Problems of Mathematics* (1987)

There are two books (rather thin books!) very worth reading for different reasons:

- G.H. Hardy, *A Mathematician's Apology* (1940; reprinted with foreword by C.P. Snow, 1967)
- G. Pólya, *How To Solve It: A New Aspect of Mathematical Method* (1945; 2nd edition, 1957; reprinted edition 1990)

3. The book you select can be written in Chinese or English, or in any other language for that matter. The essay is to be written **in English**. You should give an account of what the book says and **your own** view and reflection after reading the book. If you make use of any other publications or webpages for reference, do list them in the bibliography at the end of your essay. Take care to avoid the pitfall of plagiarism, and definitely not to commit the act.