# Forty-five Years of HPM Activities: A Semi-personal Reflection on What I Saw, What I Heard and What I Learn

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## Abstract

HPM (History and Pedagogy of Mathematics) activities deal with integrating the history of mathematics with the teaching and learning of mathematics. As a teacher of mathematics the author will share his personal experience in the engagement of HPM activities during the past forty-five years with fellow teachers who are interested in such activities and who may wish to know how another teacher goes about doing it.

**Keywords:** History and Pedagogy of Mathematics, history of mathematics, mathematics education

## 1. Introduction

HPM (History and Pedagogy of Mathematics) activities deal with integrating the history of mathematics with the teaching and learning of mathematics, which can further be refined into three interrelated aspects: (3a) learning and teaching a certain subject area in mathematics, (3b) providing general motivation and enjoyment in studying mathematics, (3c) nurturing a deeper awareness of mathematics and its social and cultural context, with a more elaborated account given in [29]. In this paper I will share with readers an account of my involvement in HPM activities, and hope that many teachers with a similar background as me may find the sharing in the account helpful.

## 2. The Beginning

My own venture in history of mathematics began with a broken leg (not mine!) Soon after I took up my first teaching post at University of Miami in the U.S.A. I was approached one summer day in 1974 by the chairman of my department who asked me to

take up the course "Introduction to Mathematical Ideas" which was used to be taught by another colleague who took sick leave because of a broken leg. This story was recounted in [24] : "With less than a month before the semester began I went through a very intensive period of studying and hard thinking to equip myself and to design a course that I thought suitable. Instead of a chore, this assignment turned out to be a blessing. […] As a matter of fact, at the time I was quite baffled myself as to the meaning of studying mathematics. Since my school days, I had been interested in mathematics. After getting my first degree I wanted to become a mathematician. I studied hard in graduate school and managed to obtain a Ph.D. by writing a dissertation on algebraic K-theory under the supervision of Hyman Bass, to whom I am forever grateful for his teaching. Then I came out to teach and, in facing a large class of students not all of whom were as interested in mathematics as I, had to think about the question, "What actually is this subject that I have been trying to study for so many years?" How could I convince my students mathematics is a meaningful subject that is worth studying? At the time I found it even difficult to convince myself of that because I then realized all along my interest in the subject was perhaps driven more by a good track record in the subject from school to university days than by an intrinsic passion. […] In enjoying my study, I never really reflected on the meaning and nature of the subject. I realized that I did not know too much about the history of the subject other than the little that I had gathered from reading a few popular accounts as a school pupil and an undergraduate. There were lots of questions on how people in the past dealt with mathematical problems that baffled me. I realized that I did not understand how mathematics came to be in the form I was then studying. This new teaching assignment afforded me a good opportunity to read up, to think hard, and to reflect carefully. At first my immediate objective was just to cope with the new teaching assignment, but gradually it dawned on me that maybe I could do more and integrate the history of mathematics into other mathematics courses that I would teach. From that first step onward, my interest in the history of mathematics increased with time so that subsequently I began to study mathematics with awe and passion for its intrinsic value." I was fortunate to have come across at this juncture the book *A Concise History of Mathematics* by Dirk Jan Struik (1894-2000) [31], which though concise (with only about 150 pages in text!) is illuminating and inspiring for me, An outline of the course I gave, as well as its underlying philosophy, can be found in [10]. This course is my first attempt of an HPM activity.

I see myself as a research mathematician as well as a teacher of tertiary mathematics, who has a deep interest in and a passion for mathematics, at the same time wishes to know more about the role the subject plays in relationship to other human endeavours and intellectual pursuit, in particular, how the subject evolved in history and in what way it interacted with other areas of culture. As a teacher I like to share with my

students this interesting story of the long quest by the human mind for an understanding of the world around us, hoping to arouse thereby their curiosity and imagination.

In [24] I say, "The history of mathematics is an academic discipline just like any other academic discipline, with its own scope of study, body of research, and literature. Although I do dabble in this area now and then, I consider myself an amateur in this academic discipline. I am not qualified as a historian of mathematics; at the most I am a friend of the history of mathematics. I am more interested in integrating the history of mathematics with the teaching and learning of mathematics. This is not the same as advocating the teaching of the history of mathematics in schools and universities. […] However, I do advocate the integration, at all levels of mathematical study, of suitable material taken from the history of mathematics to enhance and enrich our teaching and to convey a sense of history. The rationale behind my position is twofold. First of all, the basic tenet I hold is that mathematics is part of culture, not just a tool, no matter how useful this tool might prove to be. As such, the history of its development and its many relationships to other human endeavours from ancient to modern times should be part of the subject. Secondly, through my own experience in teaching and learning I have found that knowledge of the history of mathematics has helped me to gain a deeper understanding and to improve my teaching."

A critical change from feeling like a lone sojourner in this attempt of integrating history of mathematics with the teaching and learning of mathematics occurred in the summer of 1984 when I got acquainted with Otto Bekken of Agder College of Norway in a working group at the ICME-6 held in Adelaide. From him I learnt of the interest of an international community of colleagues in such activities. Four years later at the invitation of Otto I participated in the Kristiansand Workshop on History of Mathematics and Mathematics Education, and immediately became good friends with an heterogeneous group of twenty-four participants — mathematicians, mathematics educators, and historians of mathematics — from different parts of the world with different cultural or academic backgrounds, who made up a most congenial and dedicated group. In the Kristiansand workshop I presented three papers dealing with various topics — functions, heuristic reasoning, mathematical thinking — embedded in a historical context [12, 13, 14]. This memorable event was my initiation into the HPM community.

## 3. Subsequent Development

In my home base at the University of Hong Kong I offered since 1976 until I retired in 2005 a course with the title "Development of Mathematical Ideas", which is an elective for upper-level mathematics students with a moderate class size of around twenty. Because I deliberately did not set for the course a fixed syllabus nor a fixed format in teaching and assessment, it allowed me to try out freely new approaches and

new teaching material from year to year [17]. From preparing this course each year I learnt a lot and got more and more interested in the history of mathematics. In other courses on technical mathematics I taught, from beginning calculus and linear algebra to abstract algebra and discrete mathematics, I tried to integrate material in history of mathematics into the courses so as to arouse students' interest and enhance their understanding of what they were learning, usually with gratifying results. Sometimes a question from the class may lead to some fruitful study. One such instance I experienced was when a student in beginning calculus asked me how come a software in symbolic calculation fails to compute an (indefinite) integral, also called a primitive, of a given function. This led me to study the esoteric topic of integration in finite terms that began with the work of the French mathematician Joseph Liouville (1809-1882) with far-reaching ramification in modern mathematics [16].

While I borrowed ideas from history of mathematics to enrich the teaching and learning of mathematics I got interested in what can be learnt from history related to the process of proof and proving, an essential part of doing mathematics both for a beginner like a school pupil and a matured mathematician pursuing research in the subject. Many such examples gleaned from history were discussed in a book [11], and a further discussion related to the cultural and pedagogical aspects was shared in a plenary panel at the 19$^{th}$ ICMI Study on proof and proving in mathematics education held in Taipei in May of 2009 [22].

In the early 1990s I began a study on the history of mathematics education, since I felt that the past will help us to understand better and thus to improve the present teaching and learning in the classroom. An underlying rationale is that in mathematics education, besides transmitting mathematical knowledge and nurturing mathematical thinking it is essential to let students shape their own views of mathematics and attitudes towards the subject. Individuals may hold different views and attitudes of the subject, but in a community these would aggregate from whence a general prevalent view and attitude emerge, which is reflected in the development of mathematics education and in the development of mathematics as well. In this sense the history of mathematics education is a part of the history of mathematics. My study began with looking at mathematics education in ancient China, which was summarized in an account presented at the HPM Satellite Meeting of ICME-7 in July of 1992 [15, 18, 30; an expanded version based on the last two papers can be readily accessed as Chapter 6 in the book *How Chinese Learn Mathematics*: *Perspectives from Insiders* edited by J. Cai, L. Fan, S. Li, N.Y. Wong, World Scientific, Singapore, 2004], then broadened to a more general context in later years, especially through the encouragement from my friends Gert Schubring and Fulvia Furinghetti, who initiated an international working group on the history of mathematics education [1, 20, 25, 28]. I will return to this point in Section 5.

In subsequent years my interest in history of mathematic gradually extended to the close relationship between mathematics and physics, then more generally to the history of science and since the late 1990s to encompass as well the role mathematics plays in a cultural context by offering a course "Mathematics; A Cultural Heritage" open to students in all departments and faculties of my university. During the decade that I gave this course until I retired I learnt much (and still am) about the role mathematics plays in other arenas of human endeavours ranging from architecture, fine arts, literature, engineering, science, economics and finance, social sciences, and others.

## 4. The Tenth ICMI Study

An important event came up in my HPM experience when in 1997 I was asked by the late John Fauvel (1947-2001) and Jan van Maanen, both of whom I got to know in the Kristiansand Workshop, to join the International Program Committee of the 10th ICMI Study on the role of history of mathematics in the teaching and learning of mathematics that they were planning. Besides serving as the coordinator of one working group, I was invited to give a plenary lecture at the Study Conference held in Luminy, France, in April of 1998. I thought up a strange-sounding title: "The (in)complete quadrangle: Historians of mathematics, mathematicians, mathematics educators and teachers of mathematics", which was inspired by my own research interest in combinatorial designs and carries a meaning in the HPM context on which I will not elaborate here [24]. After the conference I helped in writing up two chapters in the Study Volume [5]. From the active discussion that went on during the Luminy Conference and the frequent correspondence with other members of the two working groups I took part in during the task of writing up the two chapters I further enhanced my experience in HPM.

In this Luminy lecture I raised the following questions: (1) How would the history of mathematics help in achieving the acquiring of knowledge and the nurturing of ability in mathematics? (2) Can it help directly or indirectly (by drawing attention to epistemological obstacles, raising morale, arousing interest, providing motivation, etc.)? (3) Can the use of the history of mathematics sometimes even hinder the growth of the mind, which might follow a more fruitful path if it were left to itself? (4) How would young children respond to the history of mathematics and can they appreciate the subtlety of it? (5) If good care is not taken of the subject content, will bringing in the history of mathematics help, or would it be like having champagne and caviar to go with a hamburger from a nondescript roadside fast-food outlet?

The main message embodied in the title is the suggestion of a kind of cooperation from all four quarters to mould a general conception of mathematics which is receptive to the history of mathematics. Moreover, it is also a wish that HPM activities should

broaden the partnership and seek the wisdom of other groups as well, such as historians of science, philosophers, and teachers of other subjects. In other words the HPM community would do well to engage in communication with colleagues in other areas of study as well. As said in [24], "As much as I treasure and enjoy this comfortable company, I also worry about a sustained development of HPM activities. In particular, I worry that an HPM group may become a friendly and closely knit group who talk to each other within the community but not as much to others outside of the community, essentially "preaching to the converted" as the colloquial saying goes."

One may wonder whether a "harmonious quadruple" is formed in other science disciplines amidst a similar quadrangle? My own experience as a school pupil tells me that history is naturally integrated with the content of these other subjects, like physics, chemistry, or biology, for which lessons are steeped in classical experiments, evolution (or controversy) of theories, and deeds of famous practitioners throughout the ages. Thus, pupils become aware that the subject has a history. In mathematics the same is usually not the case so that pupils only see the subject as methods in calculation that seem to be given by a all-knowing overlord. Why is mathematics like that? In [24] I try to make a guess: "Is it because mathematics has become, in the minds of many, an abstract subject of purely mental constructs, so much so that one need not care about its past and what our predecessors have done in order to make progress? And indeed, what is meant by progress if one can (seemingly) create one's own problem and solve it? It seems one can achieve in mathematics without knowing its history, and conversely knowing its history may not help one to achieve in mathematics." Great masters of mathematics thought otherwise. Niels Henrik Abel (1802-1829) said that one should study the masters and not the pupils. Henri Poincaré (1854-1912) said that to foresee the future of mathematics the proper course is to study the history and present condition of the science. In his ICM-1978 plenary address, André Weil (1906-1998) borrowed a saying from Gottfried Wilhelm Leibniz (1646-1716): "Its use is not just that History may give everyone his due and that others may look forward to similar praise, but also that the art of discovery be promoted and its method known through illustrious examples."

I further guess: "It sometimes occurs to me how much influence the development of mathematics in the Western world, growing out of the Greek period and epitomized in Euclid's *Elements*, did exert on shaping the formation of the "introversion" of mathematics mentioned above. The development of mathematics in cultures other than Greece did not follow the same pattern. Admittedly, Western mathematics has dominated the scene from the 17$^{th}$ century onward. Can we learn something from the differences in ways of thinking and philosophies in mathematics between the East and the West? In mathematics education, can we assimilate these two styles of doing mathematics, the "dialectic" and "algorithmic" [explained by Peter Karl Henrici (1923-1987) in: Computational complex analysis. *Proceedings of Symposia in Applied*

*Mathematics*, 20(1974), 79-86], which, like the *yin* and *yang* in Chinese philosophy, should complement and supplement each other?" [24]. This line of thinking led me to a comparative study of Euclid's *Elements* and ancient Chinese mathematical texts, in particular, the *Jiuzhang Suanshu* [九章算術 The Nine Chapters on the Mathematical Art].

Indeed, a more serious study on history of mathematics and science was aroused when I got acquainted with a group of historians of science and mathematics through an incidental (and fortunate) participation in the 19th International Congress of History of Science held in Zaragoza, Spain in August of 1993. One thing I learnt from talking and working with this group of historians of science and mathematics is the difference in emphases and vantage points a researcher in history of mathematics and an active participant in HPM activities will take, which, though different, can complement and supplement our HPM experience. I was further inspired by a saying of the late historian of mathematics Ivor Grattan-Guinness (1941-2014) who said that "both history and heritage are legitimate ways of handling the mathematics of the past; but muddling the two together, or asserting that one is subordinate to the other, is not. [...] the history of mathematics differs fundamentally from heritage studies in the use of mathematics of the past, and that both are beneficial in mathematics education when informed by the mathematics of the past." [6].

## 5. My HPM Activities in the Recent Two Decades

In connection with this interest I have been occupied in the recent two decades with the study of the transmission of scientific learning in general and mathematics in particular between the East and the West, with the iconic beginning of the translation of Euclid's *Elements* (the 16th century rendition by Christopher Clavius (1538-1612)) by Matteo Ricci (1552-1610) and XU Guang-qi (徐光啟 1562-1633) in 1607. In the 6th European Summer University on History and Epistemology in Mathematics Education held in Vienna in July of 2010 this story of the first transmission of a Western mathematical treatise into China was presented as a workshop [21]. Subsequently I studied the transmission of other works in European mathematics into China in the 18th and 19th centuries, in particular the collaboration of the British missionary Alexander Wylie (1815-18887) and the Qing mathematician LI Shanlan (李善蘭 1811-1882) [2, 3, 7] and that of another famous pair of collaborators John Fryer (1839-1928) and HUA Hengfang (華蘅芳 1833-1902) [8]. One other interesting geometric problem, the so-called Malfatti Problem on three circles, that came up in Europe, Japan and China in the 19th century, caught my fancy in connection with the study of this period, thereby providing a nice topic for conducting a workshop at the 7th European Summer University on History and Epistemology in Mathematics Education held in Copenhagen in July of

2014 [26]. This long story of transmission of mathematical learning between the East and the West that spanned two thousand years was presented at the HPM Satellite Meeting of the ICME-13 in July of 2016, playfully dubbed the "confluence of the Yellow River and the Mediterranean" [27].

A by-product of this study on the pedagogical side is a series of lectures under the general heading "Ancient/medieval Chinese mathematics — in the context of the school curriculum of today" I gave to school teachers from 2016 to 2018, with the first one on a general introduction, followed by the second one with a subtitle "Initial encounter of XU Guang-qi and school pupils of today with synthetic Euclidean geometry" and the third one with a subtitle "Initial encounter of Emperor Kangxi and school pupils of today with algebraic equations", and with two yet to be prepared lectures of a similar nature on the respective topics of calculus and probability theory that were transmitted into China after the mid-19$^{th}$ century in the late Qing period. A motivating factor in preparing these lectures is the similar feeling school pupils of today may experience when the subject is first introduced in their lessons, just like how the Chinese reacted to this "new learning" from the West, which was foreign to their traditional learning. As we know, what is learnt today in the school mathematics curriculum is basically what was established in Europe since the 16$^{th}$ and 17$^{th}$ centuries.

Since the mid-1990s there has been an upsurge of interest in the process of learning and teaching in a classroom environment dominated by the so-called Confucian heritage culture (CHC), which is brought into focus in the form of two paradoxes, namely, the CHC Learner Paradox and the CHC Teacher Paradox, which I would not go into here but refer readers to two books [32, 33]. In examining the official curriculum in mathematics of the Tang Dynasty (618-907) and attempting to re-construct from it possible questions set in an official examination of the time I tried to discuss the phenomenon of the CHC learners and teachers from that perspective [18, 28].

Another issue lies in character building through the study of mathematics, in particular the benefit gained from learning proof and proving. Important though this is, it is seldom emphasized in Western education, while this point had been emphasized in China since early time, perhaps as a result of the influence of the Confucian philosophical heritage. In an essay on the Chinese translation of the *Elements*, the co-translator Xu Guang-qi wrote,"The benefit derived from studying this book [the *Elements*] is many. It can dispel shallowness of those who learn the theory and make them think deep. It can supply facility for those who learn the method and make them think elegantly. Hence everyone in this world should study the book. [...] Five categories of personality will not learn from this book: those who are impetuous, those who are thoughtless, those who are complacent, those who are envious, those who are arrogant. Thus to learn from this book one not only strengthens one's intellectual capacity but also builds a moral base." We find a similar message echoing in our time, as the late Russian mathematics educator Igor

Fedorovich Sharygin (1937-2004) once said, “Learning mathematics builds up our virtues, sharpens our sense of justice and our dignity, and strengthens our innate honesty and our principles. The life of mathematical society is based on the idea of proof, one of the most highly moral ideas in the world.” [22]

## 6. What is to be Done?

In the HPM-2004/ESU-4 conference held in Uppsala, Sweden I played the devil’s advocate to offer a list of sixteen factors that may cause hesitation on the part of teachers in making use of history of mathematics in the classroom or discourage them to do so [19]. To dramatize those sixteen factors I phrase each as either an exclamation or a question, as if it is uttered by the teacher herself or himself. Here are the sixteen factors: (1) “I have no time for it in class!” (2) “This is not mathematics!” (3) “How can you set question on it in a test?” (4) “It can’t improve the student’s grade!” (5) “Students don’t like it!” (6) “Students regard it as history and they hate history class!” (7) “Students regard it just as boring as the subject mathematics itself!” (8) “Students do not have enough general knowledge on culture to appreciate it!” (9) “Progress in mathematics is to make difficult problems routine, so why bother to look back?” (10) “There is a lack of resource material on it!” (11) “There is a lack of teacher training in it!” (12) “I am not a professional historian of mathematics. How can I be sure of the accuracy of the exposition?” (13) “What really happened can be rather tortuous. Telling it as it was can confuse rather than to enlighten!” (14) “Does it really help to read original texts, which is a very difficult task?” (15) “Is it liable to breed cultural chauvinism and parochial nationalism?” (16) “Is there any empirical evidence that students learn better when history of mathematics is made use of in the classroom?”

Later on I realized any enthusiastic member of the HPM community has to come face to face with these unfavourable factors with an open and modest attitude instead of merely staying in a defensive frame of mind of the devil's advocate in order to avoid falling into the aforementioned trap of “preaching to the converted”. Apparently the paper achieves its purpose in drawing the attention of a number of authors to offer their views on these factors and to seek ways to turn the negative features to positive use, for instance, Adriano Demattè and David Pengelley gave detailed and illuminating discussion related to the use of primary sources in relationship to those sixteen factors [4, 9].

In my home base besides teaching the course on “Development of Mathematical Ideas” I help to run an annual series of seminar-workshop for local school teachers from 2010 to this date. Through this activity I become more strongly aware of how one should

examine a topic from three perspectives: a historical perspective, a mathematical perspective, and a pedagogical perspective. In [23] I explained further, "Although the three are related, they are not the same; what happened in history may not be the most suitable way to go about teaching it, and what is best from a mathematical standpoint may not be so in the classroom and is almost always not the same as what happened in history. However, the three perspectives complement and supplement each other. For a teacher, it is good to know something about the historical perspective, to have a solid idea of the mathematical perspective, and to focus on the pedagogical perspective."

This series of annual seminars came out of the hard work of members of a core HPM Study Group which was started in the summer of 2007 and met about five times a year. Initially the main programme taken on by the Study Group is to study collectively two famous treatises, namely, Euclid's *Elements* and the ancient Chinese mathematical classics *Jiuzhang Suanshu* [The Nine Chapters on the Mathematical Art]. Members of the Study Group present at regular meetings what they had studied from these two treatises. As time goes on, related topics from other sources in line with the objectives of HPM activities are also discussed [23]. The number of "stalwarts" who come faithfully to participate in these meetings is not large, just about a dozen or so, but all members relish every single meeting in which they freely share ideas and experiences. I raise my hat to this group of dedicated teachers true to their teaching profession, realizing the very heavy workload and work pressure local school teachers are placed under. "Compared to school teachers elsewhere that I have met in HPM conferences, this fledgling local group is just taking a small initial step and has a far way to go, but we are trying." [24].

It is important to promote the interest of school teachers in the role of HPM and to build up a core network of school teachers to get involved actively in HPM activities, because they are the ones who are working in the forefront --- the classroom. As said in [24]: "It is clear that the production of more didactical materials with a historical dimension ready for use in the classroom will foster the cause of HPM. The question is: "How and by whom are these didactical materials to be produced?" Passive reliance on ready-made didactical materials produced by others will not be good enough. All teachers should realize that they themselves can contribute actively. Furthermore, treating ready-made didactical materials as recipes to follow is not helpful either. Without a reasonable amount of immersion in the history of mathematics, teachers cannot really acquire the essence of it and will lack the self-confidence to integrate the history of mathematics with their teaching, especially in the company of a class of zealous and inquisitive students who may compel the teacher to leave a prepared path but thereby bring benefit to everybody in the class."

To achieve this purpose we need to support each other, particular those who begin to develop interest in this direction. There is no dearth of reference material on history of

mathematics and on HPM activities, a vast amount of which (in fact, more than one can digest!) is within ready access on the internet, including many manuscripts of primary sources as well. However, those who begin to develop an interest in HPM activities may feel not as comfortable as to where to start from. In the summer of 1999 at a two-day-in-residence workshop on HPM for school teachers I prepared a series of lectures and offered a short "booklist" for beginners, which is not meant to be exhaustive nor updated but may prove to be of some help to beginners in preparing their lessons. The majority of items in the "booklist" are in English (although the document is written in Chinese) and it can be accessed at the link:

https://hkumath.hku.hk/~mks/HistMathTeachingReadingListChinese.pdf ).

When an initial interest has developed to some extent teachers can proceed to a more intensive study of the history of mathematics, selecting topics which catch their fancy. In this kind of study it is important to try to go to primary sources, even if it calls for some hard work but will be rewarding. It would prove of great benefit to begin with a more serious study of two classics, Euclid's *Elements* in the West and *Jiuzhang Suanshu* [The Nine Chapters on the Mathematical Art] in the East, as mentioned above in the HPM Study Group.

What is to be done through the collective effort of the HPM community? Producing didactical materials with a historical dimension as mentioned above is one way. Besides producing didactical materials, suitably designed empirical studies to examine how effective such activities can be may attract more supporters. A third way is to encourage members to make records of their classroom activities including written accounts or better yet, videos. Local and international conferences may help to promulgate ideas in teaching and generate a much needed *esprit de corps*.

## 7. Epilogue

In this semi-personal account on HPM activities one will not find a clear and comprehensive theoretical framework, or an outline of the methodology, or some informative empirical results to substantiate the effectiveness of these activities. I lay no claim to novelty, for I have borrowed much from the ideas and theoretical frameworks of others. I do not mean to say that empirical studies on the effectiveness of HPM activities are not worth carrying out, Rather, I try to illustrate these points through telling the story of what I experienced in practicing HPM in my teaching during the past forty-five years.

To engage in HPM activities one has to invest time and effort to equip oneself for the task. There is no substitute for assiduous study on one's own. The author's experience is that knowledge is accumulated by bits and pieces over months and years

and is never ending. It is no easy task, but it is meaningful and enjoyable. If we do not get involved or get started, then nothing will be accomplished [24].

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