Transmission of probability theory into China

LIH Ko-Wei (李國偉)[1] and SIU Man-Keung (蕭文強)[2]
[1]Institute of Mathematics, Academia Sinica, Taipei, Taiwan
[2]Department of Mathematics, University of Hong Kong, Hong Kong, China

[Presented by M.K. SIU (e-mail: mathsiu@hku.hk) on 27 August 2013 at an Invited Paper Session of the 59th World Statistics Congress held at the Hong Kong Convention and Exhibition Centre]

I like to thank the organizers for inviting me to give this presentation, thereby providing me with the motivation and incentive for going into this study. Such motivation and incentive are needed, especially because I am neither a historian nor a researcher in probability, but just a teacher of mathematics who is interested in the history of the subject for a pedagogical reason.

This story is about how the theory of probability was first introduced into China in the late-nineteenth century. In a way this is not a story of success, but not one of complete failure either. The first translated book on probability introduced this new branch of mathematics to the Chinese mathematicians and scientists towards the ending years of the old Imperial China, but seemed to have exerted little influence on the subsequent development of probability and statistics which took place in the Chinese Republic. The serious study of probability and statistics, in the modern sense of the subject, got underway in China in the 1930s, from which point onwards the development would be like what it was in the Western world.

Why are we interested in this "unsuccessful" attempt? A main reason is to look at the episode from the viewpoint of the incompatibility in the cultural, social and linguistic context when a certain scientific discipline from outside is implanted into the indigenous environment in which the concept and technical means are foreign and the communication is not helped by the scientific language of the time.

The translated book was given the title *Jueyi Shuxue* (決疑數學), which has the literal meaning of "mathematics to resolve uncertainty". Several historians of mathematics had researched on the book with the

commonly acknowledged view that it is a translation based on an article of Thomas Galloway (1796-1851) in the eighth edition of *Encyclopaedia Britannica* in 1859 [see Bréard (2008); Guo (1989); Wang (2006); Wang, Bréard (2010); Yan (1990)]. Galloway's text first appeared as an article in the seventh edition of *Encyclopaedia Britannica* in 1839 and was published separately as a book. The translation was accomplished through the collaboration of the British missionary John Fryer (傅蘭雅 1839-1928) and the Chinese mathematician HUA Heng-Fang (華蘅芳 1833-1902) in the traditional way of those days, namely, the Western translator would explain the text in detail to the Chinese translator, who would then study the content to as much depth as possible and wrote out a Chinese text in consultation with the Western translator. The translation was published in 1896, although it was already announced in the Chinese magazine *Gezhi Huibian* (格致彙編) that the translation had been finished by the autumn of 1880, more than fifteen years before its publication.

Jueyi Shuxue faithfully reproduces the original English text. The content is basically a book on probability after the style of Laplace with the work of Poisson added in. The introduction begins with the sentence:

"The doctrine of probability is an extensive and very important branch of mathematical science, the object of which is to reduce to calculation the reasons which we have for believing or expecting any contingent event, or for assenting to any conclusion which is not necessarily true."

After some general discussion of the subject a historical account of the theory from the Pascal-Fermat discussion to the fundamental treatises of Huygens, Bernoulli and others are given including the landmark classics *Ars Conjectandi*, followed by the works of Laplace and Poisson with the work of De Morgan of 1837 mentioned at the end. By the end of the nineteenth century probability theory had developed beyond the dominance by Laplace's work, but the translation indicates the enthusiasm the translators still held for Laplace's work.

Incidentally, in two places the names of Monmort and Bayes are mentioned, more about whom we have just learnt from the presentations by Professor David Bellhouse and Professor Kai Ng in this session.

It is interesting to note a passage in the introduction that says of probability and gambling:

"In fact, most of the questions of this class to which the calculus can be applied, are connected with lotteries and games of hazard. The results obtained from the analysis of such questions cannot be considered as being of any great value in themselves, but they frequently throw light on subjects of far higher importance which present analogous combinations. It is true that the mathematical theory comes in aid of moral considerations, and demonstrates the ruinous tendency of gambling even when the conditions of the play are equal, mathematically speaking; but, unfortunately, those who indulge a passion for the vice are seldom capable of appreciating the force of such arguments."

This expression of a negative attitude towards gambling perhaps has an appeal for the Chinese translator who was brought up in a Confucian tradition!

The translation of Western books on science and mathematics went on as a fervent activity in Imperial China in the latter half of the nineteenth century. One has to look at it in the historical context of what the country went through during that period, which the historian Immanuel HSÜ (徐中約 1923-2005) describes as a "search for a way to survival in the new world that had been forcibly thrust upon China by the West after the mid-19th century". The Chinese, "burdened by tradition and heritage, and as yet ignorant of the nature of the Western world, groped in the dark". HSÜ maintains that "the dynamics of change suggests that modern Chinese history is not characterized by a passive response to the West, but by an active struggle of the Chinese to meet the foreign and domestic challenges in an effort to regenerate and transform their country from an outdated Confucian universal empire to a modern national state, with a rightful place in the family of nations".

After *Jueyi Shuxue* was published in 1896 it was reprinted in 1909 by the Chinese mathematician ZHOU Da (周達 1879-1949), who had great interest in the book but lamented that it was not given adequate attention so that circulation was not widespread. The fact that the book remained unpublished for fifteen years after it was translated may reflect the doubt of the translators about its degree of acceptance by the indigenous scientific community. Or, it may reflect the lack of confidence of the translators in having a comfortable grasp of this new branch of mathematics themselves. This may explain why the book was not prefaced by an introduction written by Fryer nor by HUA, particularly when two other well-known and popular translated books by them contained such explanatory introductions. This famous pair of collaborators produced *Daishu Shu* (代數術 Method of Algebra) in 1873 and *Weiji Suyuan* (微積溯源 The Origins of the Differential and Integral Calculus) in 1874, respectively translation of the articles on "Algebra" and "Fluxions" written by William Wallace for the eighth edition of E*ncyclopaedia Britannica*. These two texts were influential to the transmission of two equally new subjects in China at the time, algebra and calculus. In contrast the new subject of probability theory did not seem to have received the similar enthusiasm from the scientific community of the time!

A leading Chinese intellectual of that period, LIANG Qi-Chao (梁啟超 1873-1929), compiled a recommended reading list of Western books in 1896 in which *Jueyi Shuxe* was included. He mentioned that the translation was completed but the book was not yet published. He also mentioned that an article in a magazine introduced the subject but not the details. This magazine is the 1891 Spring issue of *Gezhi Huibian* (格致彙編) [literal meaning: compendium for investigating things and extending knowledge], a Chinese magazine published by John Fryer from 1876 to 1892. This article is "Using mathematics to resolve uncertainty", which was probably the only other mentioning of the subject at the time.

In one of his political essays titled "Nü xue [女學 Women's schooling]" of 1897 LIANG Qi-Chao also referred to the book *Jueyi Shuxe*:

"[…] What the Westerners in their translations called 'production and distribution' is what is expressed in the *Great Learning* (大學) with "Those who produce are numerous, those who eat are few." *Guanzi* (管子) said : "If a single peasant is not engaged in farming, someone will suffer hunger; if a single woman does not engage in weaving, someone will suffer cold." These are not empty words. The above is like taking the entire population of a state and its material production, and by the *Mathematical Art of Probability* (*Jueyi shushu* 決疑數術), increasing and decreasing, dispersing and adding it. The obtained ratio is indeed like this: in China, those who sponge on others are half of those who create profit. […] There are two hundred million women who belong entirely to the consumers and of whom not a single one is productive. Because they cannot support themselves and have to be supported by others, men elevate them like dogs, horses, or slaves, which makes women's life even more miserable."

Even though LIANG Qi-Chao referred to the notion of probability in his political essay on reform in advocating literacy for women and hence in defending women's rights, it seems that he did not entirely grasp the concept of probability and regarded probability just as simply a proportion!

This kind of superficial understanding of probability by a well-educated scholar of the time is not surprising. The actual influence of the book *Jueyi Shuxe* seemed to be rather minimal. Subsequent study and works on probability and statistics seemed not to have received direct impact from the book. It turns out that most of the terms the translators coined in probability theory did not survive so that Chinese terminologies adopted in later textbooks were different. An obvious example is "jueyi shuxue" (決疑數學), which was later replaced by "gailü lun" (概率論) after the temporary adoption of various other translated terms.

One argument for the lack of influence of the book has to do with the mathematical language adopted in the translation, which was not a complete symbolic format already commonly adopted in Western mathematical literature but was a hybrid between the Chinese format adopted in traditional Chinese mathematical text and some newly coined symbols with a Chinese flavour. In our modern eyes this

would indeed look awkward and impede facility in computation or even ease in comprehension. Let us look at some excerpts.

(1) Section II, Article 13: Application of the binomial theorem.
It deals with the expansion of the expression (p + q) to the power h.

(2) Section VIII, Article 96: Method of computing the value of the integral which expresses the probability.
It deals with the computation of the famous integral of *e* to the power minus *x* squared.

However, with sufficient practice familiarity can still be attained. Thus, it seems that hindrance arose more from a cultural and social context, to which we now discuss.

As HUA Heng-Fang might have expected the concept of probability was not as readily received by the indigenous scientific community of his time. With algebra there was the tradition of solving equations in ancient and medieval China, in fact of a level quite advanced compared to that of the Western world in the corresponding period. With calculus there was the notion of infinitesimals used in the computation of area and volume as explained by LIU Hui (劉徽) in the third century. But the quantitative notion of uncertainty was totally absent in all Chinese mathematical classics up to that point. This peculiar "miss" in the history of Chinese mathematics is a baffling question on two counts. The first one is an "internal" issue, since the needed mathematical knowledge and skill in numerical calculation had long been well developed in ancient and medieval China. The second one is an "external" issue, since probability theory has so many applications. It is commonly agreed that traditional Chinese mathematics paid much attention to demands on solving real world problems as an exemplification of a basic tenet of traditional Chinese philosophy of life shared by the class of *shi* (士 intellectuals), namely, self-improvement and social interaction (經世致用). Had the Chinese mathematicians become aware of the fundamental significance and applicability of the notion of probability they should devote their effort to its investigation. One may argue that the actual "real-life" problems in China of the nineteenth century were not yet

developed to the stage that gave rise to probability theory in the European world of the seventeenth and eighteenth centuries.

Perhaps this question has to be viewed not just in the Chinese scene by itself alone but in the full background of the West as well [see Bernstein (1996); Daston (1988); David (1962); Hacking (1975/2006);.Hald (1990, 1998); Maistrov(1974); Meusnier (1996); Plato (1994); Sheynin (1974): Stigler (1986)]. In spite of the occurrence of many uncertain events in human experience in different civilizations since antiquity, a quantitative approach to probability was not developed until the sixteenth and seventeenth centuries in Western Europe. For instance, gambling as a human activity has a very long history, an early example of risk taking. According to Florence Nightingale David "the real problem which confronts the historian of the calculus of probabilities is its extremely tardy conceptual growth – in fact one might almost say, its late birth as an offspring of the mathematical sciences". Mark Elvin points out that, according to what Alistair Crombie categorizes as six styles of scientific thinking in the European world, probabilistic and statistical thinking was the last one that came to maturity, and is perhaps the least natural kind of scientific thinking which needs time to incubate, be it in the West or the East. Shmuel Sambursky points out an interesting feature of Greek philosophy in general and its mathematics in particular that caused their "miss" in understanding the intriguing and important concept of probability.

In the Chinese scene there were likewise instances in philosophy, in mathematics and in social practices that could have hinted at probabilistic thinking, but this had not germinated and bloomed into a mathematical theory as it did in the West. No recognizable mathematical account on the notion of probability appeared in the mathematical literature in China before its transmission from the West through *Jueyi Shuxue*. Mathematically speaking it is known that Chinese mathematicians developed much earlier than Pascal what is commonly known as Pascal's triangle, solved problems related to permutations and combinations, and for administrative reasons treated problems of a statistical nature since early times.

Folklore wisdom depicted in some anecdotes embraced an awareness of chance events. In the old book *Thirty-six Stratagems* (三十六計)

the Stratagem Number 27 known as “Faked madness but not insane [假痴不癲]” tells the anecdote of how General DI Qing (狄青 1008-1057) of the Northern Song Dynasty in the 11th century boosted the morale of his army before the battle against NONG Zhi-Gao (儂智高 Nungz Ciqgau 1025-1055), the chieftain of the ethnic group of Zhuang (壯族) in the Southwestern part of China, by tossing 100 coins all with heads up! The whole army hailed at this omen of success and marched onto battle with full confidence. General DI ordered to have the 100 coins nailed to the ground and covered up by a silk cloth to await a triumphant return. The army did return in triumph, only to find out that all the one hundred specially prepared coins had heads on both sides! This anecdote indicates that people in the old days knew that the chance for certain events was slim.

In Book 18 of *Mengxi Bitan* [夢溪筆談 Dream Pool Essays], compiled by SHEN Kuo (沈括 1031-1095) in 1088, there is recorded the following problem in counting the number of possible situations in chess:

“The story-tellers say that I-Hsing (一行) once calculated the total number of possible situations [局] in chess, and found he could exhaust them all. I thought about this a good deal, and came to the conclusion that it is quite easy. But the numbers involved cannot be expressed in the commonly used terms for numbers. I will only briefly mention the large numbers which have to be used. With two rows and four pieces the number of probable situations will be 81 different kinds. With three rows and nine pieces the number will be 19,683. Using four rows and sixteen pieces the number will be 43,046,721. For five rows and twenty-five pieces, the number will be 847,288,609,443 …. Above seven rows we do not have any names for the large numbers involved. When the whole 361 places are used, the number will come to some figures (of the order of) ten thousand written fifty-two times [$10{,}000^{52}$ 連書萬字五十二] .”

One problem in *Shushu Jiuzhang* [數書九章 Mathematical Treatise in Nine Sections] of QIN Jiu-Shao (秦九韶 c.1202-1261) in 1247 made use of the idea of sampling in collecting grains for taxation:

"When a peasant paid tax to the government granary in the form of 1534 *shi* [石]of rice, it was found out on examination that a certain amount of rice with husks was present. A sample of 254 grains was taken for further examination. Of these 28 grains were with husks. How many genuine grains of rice were there, given that one *shao* [勺] contains 300 grains?" (Note: In the mensuration system of the Song Dynasty, 1 shi = 10 dou [斗] = 100 sheng [升] = 1000 he [合] = 10000 shao. According to tradition recorded in *Jiuzhang Suanshu*, a grain of rice with husk was counted as half a grain of rice.) [The answer is given as: 4,348,346,456 grains, out of the original 1534 x 10000 x 300 = 4,602,000,000 grains.]

In the late 17th century CHEN Hou-Yao (陳厚耀 1660-1722) treated in his *Cuozong Fayi* [錯綜法義 The Meaning of Methods for Alternating and Combination] problems of permutations and combinations in connection with divination (trigrams, hexagrams), the ten heavenly stems [天干] and the twelve earthly branches [地支] to form the astronomical sexagesimal cycles, dice throwing and card games.

Games of chance were never unfamiliar to the Chinese, even though gambling practice was officially banned by different rulers throughout the long history of China. Divination [占卜] and fortunetelling may lead to probabilistic thinking (although it may also lead to the opposite by thinking that human affairs are destined by the deities so that it is meaningless to investigate the notions of chance and risk!) Physiognomy [面相學] that had been practiced in China for long can be regarded as some kind of statistical thinking. In his very thorough analysis of the situation of probabilistic thinking in pre-modern China Mark Elvin points out that there may be more than what was found in writings. In another detailed and comprehensive study on probability and statistics in China Andrea Bréard suggests that further scrutiny is needed. I cannot go into the detailed discussion in a 30-minute talk but refer those who are interested to the works of Elvin and Bréard [see Elvin (2002/2010); Elvin (2004); Bréard (2008)].

Having mentioned all these factors we have to admit that in the history of Chinese mathematics the kind of probing into mathematical concepts, purely for the curiosity of intellectual pursuit, was not as

marked as in the West. The axiomatic approach of geometry is one example; the investigation of the solvability by radicals of polynomial equations instead of just obtaining highly accurate methods of solving them is another, and (probably!) the study of probability starting from games of chance is a third.

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