成文迄今，倏忽廿載。文內述及的想法與處理手法，於今天而言未必盡同，但總的基本想法仍然一樣
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Mathematics for math-haters

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This article attempts to answer the following questions about a course commonly known as ' Mathematics for Liberal Arts Students ', based on the author's thought and experience gathered from teaching such a course: (1) Why should there be such a course? (2) What do we want to get across? (3) What can we talk about in the course?

1. Prologue

In American college catalogues one usually finds a course described as ' Mathematics in the Humanities ', or ' Mathematics for Liberal Arts Students ', or ' Mathematics Appreciation ', or ' Introduction to Mathematical Thinking ', or ' Mathematics for Poets ', or . . . , which are all euphemisms for ' Mathematics for those who hate mathematics, but have to take it for a course requirement anyway '. Such a course is usually offered under the following formats, as exemplified by the numerous texts that appear: (1) as a more or less historical course, but best described as ' Mathematical Archaeology '; (2) as an assortment of popular topics such as set theory, symbolic logic, boolean algebra, graph theory, topology, computers, group; (3) as a technical course in pre-calculus mathematics, usually with a slant towards the so-called ' finite mathematics '. These formats have their separate merits as well as inadequacies. Such a course is of sufficient significance (in contrast to the view of some people which dismisses it as just an ' easy course ' for those mathematically inept) to warrant some serious discussion, in the hope of bettering the course with cooperative effort. It is with such hope that the author ventures to describe what he has done while teaching at the University of Miami. No claim is made to any novelty of idea, but it seems that not much discussion has been carried on so far in this important direction. Indeed, the author based his plan on a very old article by Ore [1].

2. Why should there be such a course?

" I am not going to be a mathematician, nor a scientist, nor an engineer, nor an accountant. Mathematics means nothing to me. I don't need it."

" Why should I care whether there is any odd perfect number or not? The world won't be better off even if we know the answer."

" What's the point of doing math? Don't tell me math helps to put a man on the moon. I know all that, but then what's so important about putting a man on the moon while millions are starving on this earth?"

These are all sensible comments about mathematics. In a sense they reveal a disturbing fact about modern day mathematics and mathematicians. We are often too absorbed in our little world that we fail to notice the bigger world around us. We have enough trouble to communicate among ourselves, and we neglect to communicate with outsiders. In recent years there has been a surge towards making the teaching of mathematics more relevant to the real world. This is certainly a good sign, for throughout history mathematics was never an isolated self-sufficient body of knowledge. (Of course, a mere emphasis on its applicability is also harmful to its healthy growth, as the contrast between the prosperous Greek era and the barren Roman era in mathematical development shows.) Yet, not much attention has been paid to its cultural value, which is perhaps more pertinent to the student body we are addressing.

Instead of haranguing further in my own inadequate words, let me quote those of Weyl:

> " We do not claim for mathematics the prerogative of a Queen of Science, there are other fields which are of the same or even higher importance in education. But mathematics sets the standard of objective truth for all intellectual endeavors; science and technology bear witness to its practical usefulness. Besides language and music, it is one of the primary manifestations of the free creative power of the human mind, and it is the universal organ for world-understanding through theoretical construction. Mathematics must therefore remain an essential element of the knowledge and abilities which we have to teach, of the culture we have to transmit, to the next generation." [2]

3. What do we want to get across?

It is a common experience for a mathematician to be told, " So you are a mathematician! Someday I must ask you to balance my checkbook. I can never do it right by myself. Ha, ha, ha!" Even though the author realizes that person is just trying to be funny and sociable, somehow it makes him wince. For one thing, mathematics is not just balancing a checkbook. For another thing, what is so funny about not being able to balance one's own checkbook? To some extent this illustrates certain common misconceptions most people have about mathematics. So, last and always, our purpose is to present an honest picture of what mathematics is about to our students. This can roughly be divided into five points.

(1) Mathematics is the accumulation of human wisdom in an effort to understand and harness the physical, social, and economic worlds. Mathematics is not just playing games with mysterious-looking, high-sounding symbols. Abstraction and axiomatization, which lie at the heart of modern mathematics, do not render mathematics simply a ' grand tautology '. We must not confuse form with content. To pick suitable postulates and to seek useful theorems, we have to exercise imagination, to exert blood and sweat, to look to intuitive experience, and to meet real-life necessities. That is what makes mathematics such a fascinating and worth-while endeavour.

(2) Mathematics is part of our culture. It has been living for 4000 years (longer than that if we include primitive notions about numbers and forms), and is still living and well. Its development usually interacts with the social,

political, economic and technological conditions at that time. More importantly, it affects and is affected by contemporary thinking.

(3) Mathematics is not a prize for a few geniuses. Maybe breakthroughs are brought about by individuals, but the results are the fruition of centuries of thought and development. Numerous incidents show that when time is ripe for a certain idea, it will come forth from several quarters. Furthermore, mathematics knows of no geographical boundaries nor of national differences. It is rather unimportant to argue who was the first to invent what, which people first discovered what, or in which book first occured the record of what. (Please note that the author does not see such a course as a training ground for historians of mathematics.)

(4) Mathematical thinking helps us to analyse problems more systematically and acutely. Reasoning is a precious human faculty which seems appropriate to be cultivated in a course on mathematics. In the same vein, some honest to goodness mathematical skill which proves useful in daily life can also be picked up in the course. Besides, that can be fun!

(5) Last but not least, every mathematician is undoubtedly awed by the beauty of mathematics when a certain idea, a certain concept, a certain method or just a certain theorem appeals to his or her heart. Nature abounds with patterns and curios which remind us of the almost magical power of mathematics. This is certainly something we must convey to our students.

4. What can we talk about in the course?

There is a lot one would like to pack into the course. Choice of material is subjective, but factors such as class time, class size, students' background, restrict it somewhat. The aim is to bring out the five points set down in the preceeding section, and the strategy is to make the stuff as palatable as possible. For the latter purpose, the lectures can be enlivened with witty remarks, jokes, anecdotes; audio-visual equipment should be exploited whenever possible; interesting films can be shown at times; homework assignments can be made amusing; cartoons can be used to decorate hand-out notes; and so on and so forth.

In what follows the author will sketch the content of a course he once conducted so as to provide a more concrete idea. The course for some 140 students is divided into two semesters with two 50 min lectures and one 50 min recitation section per week. In the first semester the approach is to survey the various ancient mathematical civilizations in a roughly linear chronological order. As the study of history should not be regarded as a study of the dead past, but rather as a study of the past to better understand the present and future, a strictly chronological order is not adhered to. Whenever and wherever possible, related modern topics are discussed to gain a better perspective. In the second semester a slightly different topical approach is adopted, centering around the important mathematical ideas that flourished in the post-Renaissance period. Admittedly there is an unsatisfactory blank in between, namely, the mathematics of the 'pre-modern-era'. With suitable comments at suitable places, such a jump should not create any impression of a discontinuous development in mathematics.

To arouse interest, the first couple of lectures are spent as an ' appetizer ', for which the author finds the Fibonacci sequence most suited. It is elementary, amusing, beautiful and best of all, is related to such far-fetched topics as musical scales, genealogical tree, plant growth, seashell formation, artistic design, probability, ... Following that, it is good to mention several real-life applications of mathematics in various disciplines to acquaint students with the power of mathematics. With such preparation we are ready to begin, and we begin at the very beginning, namely Stone Age mathematics (for which an article by Struik [3] is strongly recommended). At this point it is appropriate to spend a couple of lectures on the idea of one-one-correspondence in tally counting, and on the evolution from grouping numeral system to positional systems. Once the idea of positional system has been grasped, it will not require too much extra work to mention numeral systems in bases other than ten, in particular, the binary system. The next series of lectures covers ancient oriental mathematics in Egypt, Babylonia and China. Just to cite a few examples how one can relate antiquity with modern time, one can mention the binary nature of Egyptian multiplication, the algorithmic nature of Babylonian extraction of square roots, and the idea of interpolation and extrapolation contained in the Chinese method of excess and deficiency. Before we move on to early Greek mathematics whose nature is drastically different from earlier mathematics, it is perhaps worth the time to pause and examine the patterns of mathematical reasoning. For this purpose there is no better reference than Polya's excellent book [4]. The first semester closes with the achievements of the Pythagoreans, with a summarizing discussion on the advent of a mathematical development of such an abstract and deductive nature.

The second semester opens with Euclid's ' Elements ' which ushered in the idea of postulational thinking, whose salient features are borne out by the breakthrough discoveries of non-Euclidean geometries during the nineteenth century. Since the idea of postulational thinking is a central feature of modern mathematics, it is worth the time to describe, in a digestible form, one example of such an abstract system with various interpretations (for instance, boolean algebra). Then we move on to the invention of coordinate geometry, whose power and convenience will emerge through the explanation of simple problems in which an interaction between algebra and geometry is called into play. The epoch-making invention of calculus in the late seventeenth century is a natural follow-up. The idea of a mathematical model, coupled with powerful techniques in calculus, partially explains the ' unreasonable effectiveness of mathematics ' in natural sciences. A discussion on calculus also affords a good opportunity to raise some intriguing questions about infinity, old and new. The idea of integration lures us back to antiquity when Archimedes and Tsu Chung-Chih computed the value of π. In short, this exciting page in the history of mathematics offers much more than one can possibly pack into six or eight lectures. For the remaining lectures, the author chooses to close with probability theory, starting with its birth in mid-seventeenth century, partly owing to its amusing nature, and partly owing to its usefulness in a wide range of applications nowadays.

Apart from articles and books mentioned in the various sections, the author borrows generally rather heavily from Eves [5], Kline [6], plus some standard works mentioned in their bibliographies. Among such one should specially mention Needham and Wang [7], which is by far the most comprehensive account in the English language on Chinese mathematics. In the first part of this century western accounts of Chinese mathematics were either scarce or inaccurate. As a result, most texts on the history of mathematics tend to neglect the mathematical development in ancient China. Such a state of affairs is unsatisfactory because it means we cannot obtain a fuller perspective of ancient mathematics as a whole.

5. Epilogue

If we are really convinced by Weyl's words (in § 2), then we owe it to ourselves to go about such a course with seriousness and sincerity. We should not see our students as dummies, thereby refraining from talking mathematics with them. Indeed the author finds most of them fairly intelligent (but of course one should not expect to see them write out a proof like a math major does). They may have forgotten a lot of the mathematics they had learned, or even start by knowing very little. They are somewhat akin to our ancestors, who knew no mathematics and yet created mathematics. Why is it that we are unable to learn it then? After these millennia we may not be brighter, but we are certainly wiser (I hope!).

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