

The Power and “*Way*” of Mathematics

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[This is an introduction to a series of two guest lectures given in the Chinese University of Hong Kong as part of their course on “Appreciation in Mathematics”.]

Most people, even if they do not study mathematics, will recognize the utility of mathematics and its power in solving problems in various areas and in explaining what is happening in the world around us, not just the physical world but also the biological and the economic-financial world as well.

What endows mathematics with its power and utility? What is the “*Way*” of mathematics? How did mathematics develop into a subject as it is today? What is the nature of the subject? Is mathematics a tool, or a way of thinking, or a part of culture? Is a mathematical proof a kind of ritual observed by a certain sect (called mathematicians)? What is a proof for — verification, or enhancement of understanding, or training of the mind, or only for professional conscience? Why are we so certain that a theorem in mathematics really holds true? How do mathematicians work? Do they only do complicated calculations? Do they just follow logical deduction, or do they allow wild thinking that may not yet be justified according to logic? When a mathematician talks about the beauty and elegance of mathematics, what is meant by that? Why can mathematics, abstract and seemingly man-made though it is, explain and be applicable to so many different phenomena in the real world? Do we discover the mathematics, or do we invent the mathematics?

It would take at least a full course to just touch upon such issues, which soon become philosophical. Since one can hardly do justice to such issues in one or two lectures, we will in these two lectures try only to have a glimpse of the power and “*Way*” of mathematics by going through some selected examples. We will work through and discuss these examples in class together, presuming knowledge in school mathematics. In the second lecture we will talk more about proofs in mathematics.