AS/1853/MATH1853/1

UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS
MATH1853
Assignment 1 (Complex Numbers)

Due Date: 7 November 2014 (FRI) (17:00)

Please submit your assignment to the assignment box (4/F, Run Run Shaw Building)

1. Let $z = 12 + 5i$. Find $\bar{z}$, $|z|$ and $Arg(z)$.

2. Find $Arg(z)$ where
$$z = \frac{1-i}{2-i}.$$

3. Verify that each of the two numbers $z = -1 \pm i$ satisfies the equation
$$z^2 + 2z + 2 = 0.$$

4. Let w be a root of the equation $z^3 - 1 = 0$. Find the value of
$$\left|2(w^{1997} + w^{2013} + w^{2047}) - 3\right|.$$

5. Find the set of complex numbers z such that
$$\frac{|\bar{z} + i|}{|z + i|} = \sqrt{2}.$$

6. It is given that $\tan(\frac{\pi}{6}) = \frac{1}{\sqrt{3}}$. Show that
$$\tan\left(\frac{\pi}{12}\right) = 2 - \sqrt{3}.$$

7. Find $a, b \in \mathbb{R}$ such that
$$1 + \sqrt{3}i = (a + bi)^2.$$

8. Show that $\coth(-x) = -\coth(x)$.