

UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS
MATH1853

Assignment 2 Combinations and Probability

Due Date: 14 November 2014 (FRI) (17:00)

Please submit your assignment to the assignment box (4/F, Run Run Shaw Building)

1. In order to enter into a competition, eight people have to be divided into two groups of four each. How many different divisions are possible?
2. If two fair dice are rolled, what is the probability that the sum of the upturned faces will equal to 10?
3. Suppose that there are two boxes, one is red and the other is blue. The red box contains 5 balls numbered 1, 3, 5, 7, 9. The blue box contains 4 balls numbered 2, 3, 4, 5. One of the boxes is picked at random by tossing a fair coin. Then a ball is picked at random from this box. What is the probability that the ball drawn is ball 5?
4. Show that $P(B|A) + P(B'|A) = 1$ by using the fact that $(B \cap A) \cup (B' \cap A) = A$.
5. Suppose that A and B are two independent events and $P(A) \neq 0$. Show that A and B' are also independent.
6. In a factory, there are two machines, M_1 and M_2 , producing 60% and 40%, respectively, of the products. It is known that 1% of the products produced by the new machine M_1 are defective. Machine M_2 is an old machine and $x\%$ ($x\% \geq 10\%$) of the product produced are defective.
 - (a) Suppose a randomly selected product was found to be defective. Find the probability $p(x)$ (in terms of x) that this defective product was produced by the old machine M_2 ?
 - (b) What can you say about the probability $p(x)$?
7. Suppose it is given that $E[X] = 2$ and $Var(X) = 16$. Find

$$E[(X + 1)^2] \quad \text{and} \quad Var(2014 + 2X).$$

8. The probability density function of X , the waiting time of a bus (measured in hours), is given by

$$f(x) = 2xe^{-x^2} \quad \text{for } x \in [0, \infty).$$

Find $P(X \geq 1)$.