

UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS
MATH1853

Assignment 3 The Family of Bernoulli Related Probability Distributions

Due Date: 21 November 2014 (FRI) (17:00)

Please submit your assignment to the assignment box (4/F, Run Run Shaw Building)

1. (a) Find the probability of having more girls than boys in a family of 6 children.
(b) Find also the probability that among 5 families, each with 6 children, at least 3 of the families have more girls than boys.
2. An urn contains m white and n black balls ($m, n > 0$). Balls are randomly selected, one at a time, until a black one is obtained. If we assume that each selected ball is replaced before the next one is drawn, what is the probability that at least k draws are needed?
3. Let X be a Binomial random variable with parameters n and $0 < p < 1$.
(i) Find $E\left(\frac{1}{1+X}\right)$.
(ii) Show that $\lim_{n \rightarrow \infty} E(1+X) \cdot E\left(\frac{1}{1+X}\right) = 1$.
4. Show that if X is a Poisson random variable with mean 1, then we have

$$E(X^n) = E((X+1)^{n-1}).$$

Use this result to compute $Var(3X+1)$ and $E(X^4)$.

5. The daily demand X of certain product is known to follow the following probability function:

$$P(x) = \begin{cases} (1-p)p^x & \text{for } x = 0, 1, 2, \dots, \\ 0 & \text{otherwise} \end{cases}$$

where $0 < p < 1$.

- (a) Find the conditional probability that the demand is $m+n$ given that we know the demand is no less than m , i.e.,

$$P(X = m+n | X \geq m) \quad \text{where } m, n \in \mathbb{N}.$$

- (b) Establish a relationship between the conditional probability

$$P(X = m+n | X \geq m)$$

and the probability $P(X = n)$. Comment on the relationship you obtained.

6. Suppose the number of accidents X follows the following probability distribution

$$P(X = i) = K \cdot \frac{2^i}{i!}, \quad i = 0, 1, 2, \dots,$$

where K is a positive constant. Find K and the probability of having three or more accidents.