

UNIVERSITY OF HONG KONG  
DEPARTMENT OF MATHEMATICS  
MATH1853

## Assignment 4 (Normal Distribution and Statistical Inference)

Due Date: 29 Nov. 2014 (SAT) (17:00)

1. Let  $X$  follows the normal distribution  $N(1, 9)$ . Find
  - (a)  $P(X \leq 1.4)$ . (b)  $P(X \leq -1.22)$ . (c) Hence find  $P(-1.22 \leq X \leq 1.4)$ .
2. Find  $k$  such that  $P(|Z| \leq k) = 0.97$  where  $Z$  follows the standard normal distribution.
3. A fair coin was flipped 40 times.
  - (a) Find the probability that the numbers of heads and tails obtained are equal.
  - (b) Use the Normal approximation to get an approximation and then compare it to the exact solution.
4. Suppose each item produced by a certain manufacturer is, independently, of acceptable quality with probability 0.9. Approximate the probability (by a Normal distribution) that at most 5 of the next 100 items produced are unacceptable.
5. Hits to a high-volume Web site are assumed to follow a Poisson distribution with a mean of 10,000 per day.
  - (a) Approximate (by a normal distribution) the expected number of days in a year (365 days) that exceed 10,200 hits.
  - (b) Approximate (by a normal distribution) the probability that over a year (365 days) more than 15 days each have more than 10,200 hits.
6.
  - (a) The hourly wage of employees in a certain service industry is believed to follow the Normal Distribution  $N(40, 5^2)$  which has a mean  $\mu$  of 40 dollars and a standard deviation  $\sigma$  of 5 dollars. The hourly wage of an employee is said to be reasonable if it is between 35 dollars and 45 dollars. A group of 10 employees was selected randomly. Find the probability that more than 7 employees of the selected group are having a reasonable hourly wage.
  - (b) Later a survey was conducted by selecting a group of 64 employees randomly. The average hourly wage of the selected group of employees is found to be 41 dollars. Assuming that the standard deviation ( $\sigma = 5$ ) remains the same, construct a 95% confidence interval for the average hourly wage  $\mu$ .
7.
  - (a) A manufacturer produces LED light bulbs of mean lifetime 12 months. Assume that the lifetime of the light bulbs follows the normal distribution  $N(12, 2^2)$  with a standard deviation of 2 months. A light bulb is said to be acceptable if its lifetime is more than 9 months. A batch of 10 light bulbs was produced independently. Find the probability that there are 9 or more acceptable light bulbs in the batch.
  - (b) Later the manufacturer suspects that the mean lifetime  $\mu$  of the light bulbs is no longer 12 months. He then conducted a survey by collecting 50 random samples of the light bulbs. The average lifetime of the 50 samples is found to be 11.4 months. Assuming that the lifetime standard deviation remains the same, construct a 95% confidence interval for the mean  $\mu$ .