UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS
MATH1853
Tutorial 1

1. Let $A = \{1, 2\}$ and $B = \{1, 3, 5, 6, 7\}$.
   (a) What are $|A|$ and $|B|$?
   (b) Find $A \cap B$.
   (c) Find $A \cup B$.
   (d) Write down all the subsets of A.

2. Let $A = \{x^2 : x \in \mathbf{Z}\}$ and $B = \{2x - 1 : x \in \mathbf{Z}\}$ where $\mathbf{Z}$ is the set of all integers. A student claims that
$$A \cap B = \{1\}$$
   because if x belongs to both A and B, then we have $x^2 = 2x - 1$. Solving the equation, we can obtain one solution 1 which also belongs to $\mathbf{Z}$. Do you agree? Explain your answer.

3. Compute
$$\frac{d}{dx} \left(\int_0^x \sin(x) \cdot e^t dt \right).$$

4. Compute
$$\int_1^2 \frac{\log_e x}{x} dx$$
   by using *integration by substitution.*

5. Compute
$$\int_1^e (\log_e x)^2 dx$$
   by using *integration by parts.*

6. Given that $\sigma > 0$ and
$$\int_{-\infty}^{\infty} e^{\frac{-y^2}{2}} dy = \sqrt{2\pi}$$
   compute
$$\int_{-\infty}^{\infty} \frac{x}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} dx.$$