TUT/1853/MATH1853/2

UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS
MATH1853
Tutorial 2

1. Let $z = 1 + i$. Find $|z|$ and $Arg(z)$.

2. Verify that each of the two numbers $z = 1 \pm \sqrt{3}i$ satisfies the equation

$$z^2 - 2z + 4 = 0.$$

3. Reduce the following quantity to a real number: $\dfrac{1+2i}{3-4i} + \dfrac{2-i}{5i}$.

4. (a) Establish the identity

$$1 + z + z^2 + \cdots + z^n = \frac{1 - z^{n+1}}{1 - z} \quad \text{for} \quad (z \neq 1).$$

   (b) Use (a) to derive Lagrange's trigonometric identity:

$$1 + \cos\theta + \cos 2\theta + \cdots + \cos n\theta = \frac{1}{2} + \frac{\sin(n + 1/2)\theta}{2\sin(\theta/2)}.$$

5. Find the set of complex numbers z for which $Re(z^2) = 0$.

6. Find the set of complex numbers z for which $\left|\dfrac{z-3}{z+3}\right| = 2$.

7. Let $P = -2 + i$ and $Q = 1 - 3i$ be two complex numbers. Show that the complex numbers on the line joining the points in the complex plane can be express as

$$z = 3t - 2 + i(1 - 4t).$$

   (b) Show that the image of the line joining the two points in the complex plane under the mapping $w = z^2$ is given by

$$3 - 4t - 7t^2 + (-4 + 22t - 24t^2)i.$$