AS/2905/WKC/1

UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS
Queueing Theory and Simulation
Assignment 1

Due Date: 14 FEB. 2014 (5:00pm) [2%]

1. In a sequence of Bernoulli trials, the random variable X that counts the number of failures preceding the first success. X is a geometric random variable with the probability function

$$P(X = k) = (1 - p)^k p; \quad k = 0, 1, 2, \ldots$$

(a) Find $P(X > k)$.
(b) Find $P(X < k)$.
(c) Find $P(X$ is an odd number$)$.
(d) Find $P(X < k | X$ is even$)$
(e) Find $P(X = k | X \leq m), \quad k = 0, 1, 2, \ldots, m.$

2. Consider a random variable X such that the distribution is uniform in the time interval $(1, 5)$.
(a) Write down the PDF $f(x)$ and CDF $F(x)$ of for the random variable X.
(b) Hence find out the mean and variance of X.

3. Suppose there is a fair coin with two faces. A head represents 1 and a tail represents 2. Let X_1 be the random variable that represents the value of the toss.
(a) Find the generating function $G_1(z)$ of X_1.
(b) Now the coin is tossed two times, let X_2 be the random variable that represents the sum of the two tosses. Find the PDF $p(x)$ of X_2 and hence find its generating function $G_2(z)$.
(c) Hence show that $G_1(z)^2 = G_2(z)$.
(d) Generalize this result for the case of tossing the coin n times.

4. In a birth-and-death process, if $\lambda_i = \lambda/(i+1)$ and $\mu_i = \mu$ show that the equilibrium distribution is Poisson.

5. Consider a birth-and-death system with the following birth and death coefficients:

$$\begin{cases} \lambda_k = (k+2)\lambda & k = 0, 1, 2, \ldots \\ \mu_k = k\mu & k = 0, 1, 2, \ldots. \end{cases}$$

All other coefficients are zero.
(a) Solve for equilibrium probability distribution p_k. Make sure you express your answer explicitly in terms of λ, k and μ only.
(b) Find the average number of customers in the system.